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A STUDY OF PUZZLES WITH SPECIAL REFERENCE TO THE PSYCHOLOGY OF MENTAL ADAPTATION.[1]

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Recent researches, stimulated largely by increasing educational concern for the capacities and interests of childhood, have resulted in a heightened scientific appreciation of play activities not only as one of the supreme revelations of human nature, but also as a most fruitful field for the study of mind. An important and rapidly growing literature, psychological and anthropological, has added much to our knowledge of the history and nature of plays and games.

One form of play, that represented by puzzles, has, however, received little attention. To show the range of this aspect of play and to indicate its somewhat special relations to fundamental problems, is the chief purpose of the present study.

Its chief lines are:

1. A consideration of some of the wider aspects of play activity, and its relation to puzzles.
2. A classification and a brief analysis of puzzles, together with a description of the puzzle *quale*.
3. A consideration of puzzle-interest, as determined by examination of questionnaire data.

[1] I wish to express my very great obligation to all the members of the Psychological Faculty of Clark University for valuable assistance,—especially to President Hall for kindly and generous coöperation at all times, and to Dr. E. C. Sanford for the suggestion of the topic, and for advice and criticism in the working out of it.

4. An experimental study of typical modes of mental reaction; (a) as determinative, in part, of the puzzle *quale*, but chiefly (b) as supplying a basis for the comparative study of the natural logic of children, men, animals.

5. Some practical applications of the foregoing.

I.

The Biological and Psychological Import of Play. What is play? Is it meaningless outflow of energy; or may it be of serious import for the life of the animal?

Two chief theories are in the field. One, that of Schiller and Spencer, interprets play as an expression of the overflow of energy. Movements are indulged in because there is a surplus of energy which must somehow be expended. Whatever useful practice and experience may result from this random activity is in a sense accidental. This view has long held a prominent place in psychology, and has certainly given important emphasis to the physiological prerequisite of play. But it is at best only a physiological theory. It fails to explain adequately why plays should assume this and that form, why these forms should vary so considerably among different species of animals and races of men.

As supplementing the Schiller-Spencer physiological view with a biological explanation, the theory of Prof. Groos [1] marks an important step in advance.

While recognizing the importance of abundance of energy as a precondition, Groos denies its universal validity, citing instances of animals which play, even when fatigued almost to exhaustion. Play is the expression of an instinct, whose teleological import is discoverable in most of the movements of animal or child. Play is a "*Vorübung und Einübung*" of activities which soon are to be necessary to the serious life of the adult animal. Of course the teleology of play is not conscious in the young. Its immediate motivation is pleasure:

1. Pleasure in the satisfaction of instinct.

2. Pleasure in vigorous activity, as such. This certainly has high value. Wundt considers it the chief motive in many forms of play. The heightened circulation of the blood, and the like, may be conceived to increase the excitability of the brain, and thus facilitate formation of new association paths.

3. Joy in "being a cause." This introduces an emotional element. Preyer emphasizes it as important. Lessing

[1] Karl Groos: "*Die Spiele der Thiere*," Jena, 1896.

thought it gave the child a deeper sense of his own reality. Not only do children exhibit this delight, but according to Groos, the kicking of a stone, the striking down of weeds with a cane by an adult in the course of a pleasure walk, may be interpreted as the expression of a pleasure in producing change.

This "*Freude an Ursache-sein*," in last analysis, may be pleasure in triumphing over obstacles, the joy in success, victory. Nietzsche translates Darwin's "struggle for existence" into a struggle for power, contest for dominion over surroundings. Its first expression in the young infant is in its attempt to master its own body. So joy in the overcoming of difficulties may be the ultimate and ripest motivation of these activities.

Many movements of the young thus represent a kind of experimentation. The stretching out and drawing in of limbs, seizing, clawing, scratching, gnawing, trying the voice, lifting and letting fall of objects, tearing, pulling, are some of the animal movements of this sort, many of which also appear in children.

It is of central importance to note that most of these play instincts are not highly specialized at birth. Most are more or less general and rudimentary forms, which may be largely modified by experience.[1] The significant point is that they furnish the initiative, at least, for certain activities essential to the life of the organism. They, together with imitation, thus protect the animal until intelligence is ripe enough to mediate proper adaptations. They also prepare the way for intelligence by leading the animal to exercise itself in many ways, and thus acquire a large repertoire of coördinations which intelligence may later utilize; for intelligence and reason cannot utilize any "material" which has not already been a part of the conscious experience of the individual. As is well known, no act can be accomplished voluntarily whose elements, at least, have not already been performed without volition.

The educability of the animal, that is, its adaptability to new conditions, obviously would be decreased by too large a number of instincts of a highly elaborated and specialized kind. Therefore these general play instincts and imitation must have fair field in order that the creature may be most responsive to new conditions. So important is individual

[1] "*Alle Jugendspiele beruhen auf Instincten. Diese Instincte sind nicht so vollkommen, nicht so sehr in allen Details dem Gehirn eingegraben, wie sie es sein müssten, wenn sie sich erst im Ernstfall äussern würden; dafür treten sie aber schon in der Jugend auf und können in Folge dessen durch Uebung noch rechtzeitig ausgebildet werden.*'" P. 74.

experience for the elaboration and adaptation of these instincts that Groos declares there must be childhood in order that there be time for play.[1] "*Die Thiere spielen nicht, weil sie jung sind, sondern sie haben eine Jugend, weil sie spielen müssen.*"[2]

To Groos, plays thus face the future. They are a definite preparation for the serious life of adult years. Moreover, as a preformist, he would even deny that any of these activities are due to the inheritance of the results of the conscious activities on ancestors. Without entering the discussion of Weissmannism *vs.* Lamarckianism, suffice it to say that some recent writers,[3] as well as earlier ones,[4] insist upon the "lapsed intelligence" view of the instincts involved. In this view, plays may be reverberations of activities of ancestors. To choose a single instance: the fighting plays[5] of boys, and indeed the whole round of activities of a certain period which mark the average youth, for a time, as a "young savage," find their readiest explanation not in relation to a serious life of the present, but in the fact that the individual is a recapitulation of the race, and that the activities in question are such as primitive man engaged in with full consciousness and definite purpose. The implications of the "reverberation" view might seem to threaten the prospective-reference aspect of Groos's theory. The dispute, however, concerns the origin of instincts, and given the instincts both parties may well agree in their propædeutic value.

Many psychologists seem to assume that certain special instincts have value for the individual, in that they are ephemeral. They rise, and under favorable conditions stimulate somewhat the unfoldment of the organism and decline. Their chief value, indeed, lies in the fact that after making their contribution to the enrichment of "soma" or "psyche," they fall away, and leave either more valuable habits, or plasticity in possession.

Nothing seems more certain than that the congenital inheritance of the child, rich as it is, does not provide sufficiently for the exigencies of his life.[6] Just this distinction marks him off from the young of the lower animals. The

[1] *Ibid.*, p. 68.
[2] Introduction, p. 6.
[3] "A Study of Fears," G. Stanley Hall, AM. JOUR. PSYCH., Vol. VIII, p. 2.
[4] G. H. Schneider: "Der Menschliche Wille," Berlin, 1882, p. 68. G. H. Lewes and E. B. Tylor also.
[5] "Teasing and Bullying," F. L. Burk, *Ped. Sem.*, Vol. IV, p. 3.
[6] The child may, as James says, have a larger absolute number of instincts than lower animals. However that may be, *relatively* to the needs of his life, the child is less adequately provided for.

latter come into the world with a relatively highly developed instinctive equipment. And in the words of Preyer "the more kinds of coördinated movements an animal brings into the world, the fewer he is able to learn afterwards." In the child "the equipment of the lower animals is replaced by the plasticity for learning by consciousness."[1] Learning must be provided for through instincts of the more general sort, as imitation and those of play; or through intelligence; or through both instincts and intelligence. For Baldwin, the play instincts are not alone sufficient, and for Groos imitation is not. Certainly the acquisition of the mental legacy involves more than either,—intelligence.

Play and imitation, as yet, have been considered only in so far as they enable the individual to acquire the simple elements of his inheritance. Even here there may be need of invoking intelligence. But man is inventive as well as acquisitive. Intelligence "grows from more to more," if not phylogenetically, certainly from infancy to maturity in the individual.[2]

Assuming as the criterion of mentality "the pursuance of future ends and the choice of means for their attainment,"[3] intelligence may be regarded as a variation which enables the organism to make better adaptations. Its neural prerequisite is plasticity. But efficient plasticity is possible only through wide variety of experiences. The modification of the nervous system, produced by experience, exhibits two opposing tendencies: one conservative, the other radical; one making for automatism, the other for variability.

The tendencies which make for mechanization include the arsenal of specialized instincts as well as the simpler forms of imitation. The echolalia of children exhibits the tendency of the circular form of reaction, in its simpler aspects, to produce the reiteration which makes for habit. Mere variety of experiences raining in upon the nervous system is not enough to preserve plasticity. In the teeth of this gravitation toward fixedness, there must be a spontaneity to take advantage of this variety. The animals most gifted with experimenting tendencies, those which utilize changes in environment most promptly, will be favored in the struggle for existence.[4]

The case is similar with man. Granting that an advanced

[1] Baldwin: "New Factor in Evolution," *Am. Naturalist*, July, '96, p. 540.

[2] Morgan: "Habit and Instinct," pp. 343 ff. Gladstone, Benjamin Kidd and Edward Bellamy are quoted as disbelieving in a progressive increase in mental capacity in the race within recent times.

[3] James, Vol. I, p. 8.

[4] Cf. H. R. Marshall: "Consciousness and Biological Evolution," Part II, *Mind*, Oct., '96.

civilization demands a degree of adaptiveness for which inheritance cannot adequately provide through specialized instincts, granting also that mental adaptation, as such, constitutes a distinctive feature of life in a highly organized and rapidly changing social environment, the play activity assumes the double function of furnishing exercise both to those instincts which represent the relatively unchanging core of life, and to intelligence itself.

Let it be called a general impulse or general instinct to exercise the intelligence as such.[1] Such a gymnastic must consist in the most widely various sorts of activity, a deployment as far as possible of all resources of body and mind in ways which are to be of use later. Those races and individuals that feel strongly the impulse thus to deploy the intelligence and exercise the "mental muscles," and find it a joy in itself, must be favored in the struggle for existence. The play instinct thus marks its possessors as the bearers of those qualities which guarantee the continued growth of science, invention, and civilization.

The classification of plays and games, by Johnson,[2] well exhibits how they shade by almost imperceptible gradations from the forms which exercise the muscles, up to those in which the mental gymnastic predominates. Of puzzles, the majority are stimuli to intellectual effort, and thus one of the various forms of this propædeutic activity.

Many plays and games, also the more serious occupations of children, may draft off this spontaneity, and disguise it under forms of action for practical ends. The impulse, cannot well at any stage be entirely "blind," and the original motives may be greatly elaborated and modified. Instead of the mere pleasure in activity continuing to furnish the sole motive, joy in being a cause, pleasure in triumphing over difficulties may become prominent. One aspect of the latter must be the feeling of rivalry. Many social games of higher

[1] Lloyd Morgan would call these highly complex and indefinite coördinations "impulses," restricting the word "instinct" to the more definite coördinations. But in spite of this narrower use of instinct, Morgan admits that some instincts are only relatively definite (*Nature* 18, A, p. 95). Marshall (*Mind*, N. S. 5, p. 380) insists that "impulse" connotes a subjective condition, and that instinct viewed as unconscious action for ends, must be expanded to cover all complex actions which bear such mark, no matter how variable may be the coördinations. "Paternal instinct," "maternal instinct," are highly complex, and represent highly indefinite coördinations, many of which are unpredictable. Nevertheless they *are* instincts. So in the latter sense have we ventured to use the word "instinct" in relation to the mental tendency discussed above.

[2] "Education by Plays and Games," G. E. Johnson. *Ped. Sem.*, III, 1.

grades are largely motivated by this spirit of competition, and even in puzzle activity it sometimes appears. But it seems probable that there is in all play activity a residuum of pure delight in activity as such. If the competition motive were the sole incentive to play activities, its explanation on some association hypothesis, as an acquired interest, might be available. Such a view, however, would need to explain why such activities should be made a basis of rivalry unless they first possessed a certain inherent interest.

Restricting the discussion to puzzles, the following are offered as some of the evidences of the existence of the instinct in question. Most people who really enjoy puzzles say they solve them simply for the sake of learning how. Many specifically deny the existence of competition as an incentive, and many prefer to work alone.

Furthermore, in spite of the mental strain and dilemma produced by a puzzle,—an unpleasurableness, which with some persons may become strong enough to produce nervous distress and marked aversion,—the mind is again and again attracted and engaged. Many persons report a distaste for puzzles, but say they find themselves drawn to them, and once having begun, find difficulty in letting go. It is hard to believe, however, that the difficult as such, without relation to possible practical experience, can be uniformly attractive. It seems rather that the difficult as represented by games and puzzles is a special aspect of the baffling which epitomizes and typifies most of the intricate forms of reaction which are met with in the actual experiences of the individual and which perhaps have been important in the history of the race. This, of course, implies a definite propædeutic function. Whether definite or only general, and whether or not the difficult, as such, is alone sufficient under any circumstances to stimulate human interest and excite intellectual activity, the above facts seem best explained by the assumption of a general intellectual play instinct or impulse.

Summary. Intelligence is no exception to the law of exercise. Just as those animals, which by fortunate variation were born with a tendency to indulge in preliminary exercise of those activities which were to serve the serious ends of adult life, were favored by natural selection, and were able to transmit such advantage in the form of general play instincts, so in a more special way those creatures, endowed with the strongest tendencies to exploit the intelligence, may have perpetuated this superiority as a general intellectual play instinct.[1]

[1] It need hardly be explained that the designation of the above tendency as "intellectual" disregards the old tripartite division of

Classification.

So far as known, no attempt to classify all varieties of puzzles has hitherto been made, and even those writers who have sought to reduce to some order the multiform mathematical recreations have failed to find an adequate scheme of classification, while those who have attempted to arrange mechanical puzzles have fared no better. These failures are not strange when it is remembered that many puzzles are "*sui generis*." Others possess characteristics which require their classification in several distinct rubrics. In the present study the systematization is of the most tentative nature and is merely designed to indicate the wide range of puzzle materials and to emphasize the more important types. The groupings will therefore often be somewhat arbitrary.

The chief groups are: Language and Word; Mechanical; Mathematical; Logical and Philosophical; Dilemmas of Etiquette, Ethics, etc.

Language and Word Puzzles.

One of the most primitive groups. The most prominent varieties are:

The Riddle:—A question usually describing the object in question in a paradoxical or ambiguous way.

The riddle which is reputed to have caused the death of Homer, and which is still current in Brittany, is as follows: "What we caught we threw away; what we could not catch we kept."—Lice. Another, found in both France and Germany: "Lives without a body, hears without ears, speaks without mouth, to which the air alone gives birth."—Echo.

The Rebus (literally by things):—Generally a riddle, part of which is expressed in pictures or symbols. Sometimes the Rebus is little more than interpretation of pictures or symbols, the content being in no other way ambiguous.

The Conundrum:—Usually a riddle, where the play is upon words rather than upon things. The solution often turns upon a pun: "Why is O the noisiest of the vowels? Because all the rest are inaudible."

The Enigma:—More poetical form of riddle. Often involves, also, a play upon words: One of the most excellent of this category is the following by Schiller, on the *Rainbow*:

"A bridge weaves its arch with pearls
High over the tranquil sea;
In a moment it unfurls
Its span unbounded, free.
The tallest ship with swelling sail
May pass 'neath its arch with ease,
It carries no burden, 'tis too frail,
And when you approach, it flees.
With the flood it comes, with the rain it goes,
And what it is made of, nobody knows."

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mind into Cognition, Feeling and Will. Intelligence is conceived to be not merely cognitive, but all these. Later in this paper we hope to show how voluntary ability is fostered and developed through the demand of these activities for voluntary attention and persistence in general. Even the moral nature thus becomes involved to a degree.

The Charade:—Often nowadays used synonymously with "Enigma." More properly it is usually a series of riddles or enigmas, each of which has reference to a single letter, or syllable of a word, or parts of a compound word, the whole series taken together referring to some word-total. Archbishop Whately is credited with the following charade: "My *first* is equality, my *second* inferiority; my *whole* superiority."—Peerless (Peer-Less). The following by Charles James Fox has the metrical form:[1]

> My *first* is expressive of no disrespect,
> But I never call you by it when you are by;
> If my *second* you still are resolved to reject,
> As dead as my *whole*, I shall presently lie."—Herring (Her-Ring).

There are many minor variations of the charade—some of more juvenile form—as, "My *first* is in lamb, but not in sheep; My *second* in shallow, but not in deep," etc.—where each line refers to a single letter. Another species, also, of great popularity among children is that in which action enters. Many of the objects are suggested by gestures and other expressive movements, by costume, and the like, thus bearing some resemblance also to the Rebus. The dramatic element appeals powerfully to the imagination, and hence is the earliest form of the charade which interests children.

Word Squares, Diamonds, etc.:—A series of riddles the answers to which are words which are to be arranged so as to form certain geometrical figures, as squares, diamonds, etc. Here an element of geometry of situation may also enter. Furthermore, induction and deduction of very definite sort are demanded. If one of the riddles be guessed outright and the word put in its proper place in the scheme, it often serves as a basis for the deduction of the remaining words. This type is too familiar to need illustration.

The Acrostic is similar to the Word Square, but usually is more difficult because only the initial letters of the discovered words are used to spell the required answer. The Double Acrostic uses both initial and final letters to spell two words. This latter form has had a special vogue in some quarters, notably in Ireland. A collection entitled "Dublin Acrostics" was published a few years ago. The Acrostic is also so prominent in puzzle columns of periodicals as to need no exemplification here.

Logogram, Metagram, Decapitations, Curtailments, Retailments are a few varieties of the puzzle wherein the word in question is made to undergo various changes. Macaulay, Fox, and William Pitt have written classical examples of the above types. The following by Macaulay illustrates the metrical type of the Logogram, although poetical form is not necessary:

> Cut off my head, how singular I act!
> Cut off my tail, and plural I appear!
> Cut off my head and tail—most curious fact!
> Although my middle's left, there's nothing there!
> What is my head, cut off? A sounding sea!
> What is my tail, cut off? A flowing river!
> Amid their mingling depth, I fearless play,
> Parent of softest sounds, though mute forever.
>
> Answer: Cod.

Hidden Words, and the like, are somewhat similar to the above group. Altogether there are more than thirty species of language and word puzzles. It is obvious that many of these not only challenge ingenuity and involve the logical processes, but also have point as information tests.

In most of the puzzles above described, the riddle element persists, however complicated by logical, verbal and other conditions.

[1] "Cassell's Book of Sports and Pastimes," p. 953.

Mechanical.

This class includes almost all the puzzles of commerce. They are denominated mechanical[1] because of their more or less substantial construction (being made of wood, iron wire, etc.), and present the conditions of the problem in tangible and portable form. Hoffman, whose book includes the most extensive treatise on the puzzles of commerce, describes upwards of 140. The main lines of his classification are followed in this section.

1. *Those puzzles dependent on dexterity and perseverance.* This group forms an exception to the definition of a puzzle as "a demand for an intellectual adjustment." While discrimination is indeed to a considerable extent involved, the main requisite is a nice coördination of muscles. Among the representatives of this group is the familiar "Pigs in Clover."

2. *Those dependent upon some trick or secret.* This is also a low type of puzzle, but requires more ingenuity and resource than the foregoing class. As examples here, are magic money-purses with hidden lock, money-banks and snuff-boxes. In these the illusion of impossibility is strong, and the secret is usually discovered only after repeated trials and in the most unexpected places.

3. *Physical Puzzles.* Most of these involve unique applications of well-known physical laws, as those of motion and gravity. Accordingly, some of the best of this class can be performed with very little apparatus, and in consequence are not fully represented among the puzzles of commerce. These are classified here because they represent a certain degree of advance in mental difficulty over the preceding group. More familiar examples of this sort are "the blowing of a small cork into a bottle"; "removing a napkin from beneath an inverted goblet of water, without moving the goblet or disturbing the water." The totally unfamiliar action of physical laws perplexes and baffles.

4. *Dissected or Combination Puzzles.* Includes all materials so constructed that from given fragments other figures or designs are to be made. All "cross" and "square" puzzles, as well as forms analogous to dissected maps, where materials and not simply diagrams are used, are included. The characteristics of this class will be considered under the head of Geometrical Puzzles. The geometrical imagination seems the chief faculty exercised.

5. *More complicated and elaborate puzzles.* A somewhat miscellaneous group. Many are constructed of wire—others of rings and loops of cord. The usual task is to separate links or remove rings from loops of cord, and the like. They represent a distinct advance in complication over the foregoing types. The element of illusion is often strong. Some appear entirely impossible. They require procedure so unaccustomed that the individual is unable to picture the solution. They are designed to baffle the visual, and chiefly the geometrical, imagination.

As early suggested the above classification makes no claim to completeness. It only serves to pass in review, and to characterize roughly the puzzle materials in question. Mechanical puzzles of

[1]Hoffman: "Puzzles Old and New," N. Y., 1893.

still higher order are also mathematical, and hence will be considered in the latter group.

Mathematical.

Here will be followed the classification of the mathematicians. "After the recreations which depend uniquely upon number, come those in which position is concerned. After the problems involving number and position, should come those recreations into which movement enters."[1] Leibnitz's suggestion offers a practical basis for the grouping of a large majority of mathematical puzzles, and in lieu of a better scheme it will be followed in this section.

1. *Numerical Puzzles.* Chiefly arithmetical. Bachet and Ozanam, whose works are the chief sources, offer a large number of these numerical and arithmetical problems. Nearly all play, either upon the application of unfamiliar properties of number, or else derive their puzzle quality from the fact that their solution is possible only through higher mathematics, whereas the ordinary man tries to solve them by means of arithmetic, and hence fails.

Some of the most famous of these puzzles are as follows:—1. To find a number selected by some one; 4 solutions. 2. "To find the result of a series of operations performed on any numbers (unknown to the questioner) without asking questions." Others partake less of the trick-nature than the above, as: "What number which divided by 2 gives a remainder of 1; divided by 3, remainder of 1; divided by 4, 5 or 6, remainder 1; but divided by 7, no remainder?" Also problems of fractions dealing with queer legacies and the like, abound. Some contain little of the real puzzle quality, but it is difficult to draw the line. Puzzles involving arithmetical progression form a rather distinct class. Some are mechanical puzzles, as the Tower of Hanöi and "Cardan's Rings" (better known as Chinese Rings). The puzzle quality depends upon the fact that no novice will appreciate that they involve progression, but will proceed in a simpler way.

Geometrical. All these problems deal in a special way with form and position, and the larger proportion also with number.

1. *Dissected and Combination Puzzles.* Nearly all geometrical. A given figure having been cut up into various segments, the experimenter is required to rearrange the fragments so as to form another figure or figures of different character. As already mentioned, the dissected map game and the well-known Richter building block games are analogous to these. There are perhaps fifty puzzles in this group. A single instance:—"Given a Greek cross of card-board. Required by two straight cuts so to divide it that the pieces when united shall form a square." The draft on discrimination, imagination and constructive ability is considerable. Sometimes the problem is purely synthetic; from given fragments, a figure of a specified design is to be constructed. The fragments are often bizarre, and the usual modes of conceiving the figure avail little. In short, the whole procedure is set contrary to habitual modes of imaginative construction. Many of the familiar puzzles with matches also belong here.

Another small group so emphasizes certain aspects of the puzzle

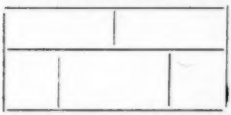
[1]Leibnitz's Letter to De Montmort, July 29, 1715.

quale as to deserve passing mention. One of the most prominent of these is as follows: A rectangle containing 64 small squares of equal size, is to be cut and reconstructed as a rectangle of different dimensions, and containing 65 apparently equal squares instead of the 64. This problem, in a way, gives the key to the paradoxical quality of most of the puzzles of the group. These paradoxes involve the element of illusion, and that in turn depends almost entirely on the inability of the eye to compare correctly the dimensions of figures where their relative position is changed.

2. *Statical Puzzles of Position.* Some of these are not clearly marked off from those of the foregoing group. The element of position is here the distinguishing feature. Number, also, frequently enters. Points, cards, counters are to be grouped in difficult combinations; as, for example, the sixteen court cards are to be arranged in a square. No row, column or diagonal is to contain more than one card of each suit and rank. Tesselation problems, map-painting puzzles, where, with a small number of colors, all the counties of a state are to be so painted that no two contiguous counties shall have the same color. Also many very familiar problems current in advertising literature, as: "How may a gardener plant eleven trees in such manner that they shall form twelve rows, with three trees in each row?"

3. *Dynamical Puzzles of Position.* These involve movement as well as position, and include nearly all the puzzles with counters, all chess and checker puzzles, railway switch and ferry-boat problems, the 14-15-16 puzzle, and many others. The "ferry-boat" problems come down from Bachet and even earlier writers. Of several varieties, the following is one of the simplest: "The captain of a company of soldiers comes to a river. The only means of transit is a boat wherein two children are paddling about, and which is so small that it will only hold the two children or one grown person. How is the transit to be effected?"

4. *Unicursal Puzzles.* A given figure is to be traced without retraversing any part of the route. Mazes, labyrinths, the knight's tour on the chess-board are examples. By formulæ the mazes may be analyzed, and the correct starting-points, as well as the minimal number of strokes necessary to trace the whole figure may be determined beforehand. A well-known simple case of maze is given on the margin. Some have extremely elaborate and tortuous forms.



Logical and Philosophical Puzzles permit of little classification. For the reason, also, that they cannot well be treated apart from their historical setting, they are included in the Appendix within the appropriate historical sections. For somewhat similar reasons Dilemmas of Etiquette, Strategy and Ethics do not appear in this scheme of classification. See Appendix.

The foregoing survey suffices to show the wide range of puzzle materials. It may be safely said that puzzle interest has levied tribute in every domain of culture material.

What marks distinguish a problem as a puzzle, and what is the "puzzle" state of mind? Each of these questions obviously involves the other, and hence both will be consid-

ered in this section. De Morgan defines a paradox as "something which is apart from general opinion, either in subject matter, method or conclusion."[1] A puzzle may be defined as a problem which is apart from the usual experience of the given individual either in subject matter or method. The method, however, is the relatively more important trait. Any problem which fulfills these conditions and which is tried chiefly for the sake of the reaction, and for the solution as such, may be a puzzle. It follows as a necessary corollary that the puzzle *quale* is in part a function of the experience of the given individual. To a mathematician armed with the principle involved, no new form of unicursal problem can possess more than a minimal puzzle element. As previously stated, nearly all puzzles make chief draft upon the more intellectual capacities. And herein lies the possibility of the introduction of every resource by which the mind may be confused and perplexed.

Some are puzzles by sheer superlativeness of intricacy. They bewilder through the multiplicity of alternatives presented. The "*Umfang*" of consciousness is not great enough to take in all the conditions necessary at any stage of the procedure. The complexity of conditions is rendered still more serious through modes of construction, which prevent the usual foresight of the end, and often, also, throw considerable difficulty in the way of mentally registering the steps already taken. A familiar instance is the recent mechanical puzzle "16 to 1." The edges of an almost closed ring are to be worked to the centre of a disk which contains a labyrinth on either side. While the subject is intent upon planning the next move on one side, the difficulties on the opposite side drop from memory, along with the whole plan of this obverse side. The puzzle thus hampers both retrospection and prevision.

Most of the possible devices which may produce *illusion* are found among puzzles. This is a distinctive feature of many. Of the rich variety of modes of this illusion quality, a few are as follows: *Geometrical illusion.* Where an unusual manipulation of a figure produces an effect simply because the geometrical imagination is not able to picture the exact consequences. The illustration, already mentioned, of cutting a rectangle containing sixty-four small, equal squares, so that it may be reconstructed into another rectangle containing sixty-five equal squares. *Illusions of impossibility:*—A wire ring is placed over large wire loops and presented in such a way that the removal of the ring seems out of the question.

[1] "Budget of Paradoxes," Introduction.

Manipulation of the wire loops may quickly modify the general form sufficiently to dispel the illusion. *Illusions of simplicity:*—The task is made to appear much simpler than it is in fact. The result is that the experimenter begins with too little consideration of the conditions involved. In many puzzles the possibility of success or failure is decided by the first step in the procedure. Apparent simplicity, therefore, leads to a premature reaction, and once drawing the mind into the maze produces a perplexity often beyond extrication. This type of puzzle seems especially effective for persons of "explosive type" of will. *Suggestion of false method:*—Of course mere intricacy may have the effect of so clouding the issue that the important features are not easily discoverable. But more definitely to produce this result, many puzzles suggest at every turn a *modus operandi* which always leads to failure. The real method is often veiled with consummate skill. As simple illustrations, may be mentioned arithmetical puzzles which really demand algebraical treatment; many mechanical and labyrinth puzzles.

Search for a general psychological explanation of the above facts brings one to the consideration of the nature of mental reactions. Modern theories of apperception have emphasized anew the truth that the mind acts in more or less habitual ways. It tends to follow lines of least resistance. There are varying degrees of elasticity of these mental habits. At one extreme are what may be termed mental reflexes—with fixed range of routine—analogous in definiteness to spinal cord reflexes. These correspond to certain uniformities in the experience of individual and perhaps of race. At the other extreme are generalized habits with fixed but wide range of plasticity. Witness the habits governing the use of syntax, and the like. These correspond to a considerable variety in the general uniformity of experience. When the mind is confronted by the new, what happens? Those ideas and remembered adaptations which seem to be most closely related to the new phenomenon are called up by association. The law of analogy holds here. Now, if the new suggests analogies to *many* different complexes of ideas, the process of assimilation is checked until one or another complex becomes prepotent. Finally, if the phenomenon is so constituted that it not only elicits many different habits, but also emphasizes unimportant features of its own content as essential, we have the discharge of a reaction which temporarily clears the field. But such an inadequate response usually multiplies the former difficulties, and so there is a new strife of tensions, more painful deliberation, which in time may be eliminated by the discriminative selection of the appropriate

mode of procedure. The progressive discovery of more and more important and vital resemblances or analogies between the new and old, is of course one of the most significant factors in producing successful adaptation.[1]

This is exactly what happens in the puzzle. The individual brings to bear reactions which are not adequate. On the one hand the old habit is attempting to reduce the new to terms of itself. On the other hand the new is waging war on the fixed, the habitual, and thus makes for new adjustments and a higher degree of plasticity. More definitely, many typical puzzles are thus designed, first of all, through wide or simply perverse suggestiveness, to "swamp" the mind with the multiplicity of more or less incongruent trains of ideas and motor tendencies called up. The dice seem loaded against the normal exercise of what James calls "sagacity"—the ability to pick out the essentials and ignore the accidents. For it is obvious that "sagacity," everything else equal, is potent inversely as the number of alternatives presented. Complexity may quickly reach a point where it is impossible for even a Newtonian mind to single out the essentials. Mind, as we know it, "cannot drive the whole universe abreast," but must take it tandem.[2]

To summarize briefly:—In spite of wide diversities of mental experience of human beings past and present, there is also evidence of great uniformity. Every creature shares to a degree in such uniformity, for all inhabit, roughly speaking, the same world. Uniformity and reiteration of experience make for habitual modes of response. The facts of association of ideas and of perception express these uniformities of mental deportment. These fundamental modes

[1] As this is intended only as a rough sketch or scheme of the processes, no account is taken of the processes of judgment, and the like, which certainly enter with deliberation, but which act only on materials which association offers.

[2] Since writing the above my attention has been called to the following from Mach: "A considerable portion of mental adaptation takes place unconsciously and involuntarily under the natural guidance of the facts presented to the attention. If this adaptation has become sufficiently comprehensive to embrace the vast majority of the occurring facts, and subsequently we come upon a fact which runs violently counter to the customary course of our thought without our being able to discover at once the determinative factor likely to lead to a *new* differentiation, then a *problem* arises. The new, unusual and marvelous act as a stimulus, which irresistibly attracts the attention. Practical considerations, or even bare intellectual discomfort, may engender a volitional frame of mind requiring the removal of the contradiction, or a consequent new mental adaptation. Thus arises *purposive* thought, adaptation, *investigation*." "Analysis of the Sensations," Chicago, 1897, p. 159.

of adaptation in men and animals, may be conceived to have survived because, on the whole, more successful than other possible modes. As has been shown, it is not necessary to assume that they have been thoroughly adequate to all experiences with the new. Indeed, if such adequacy existed, the history of human perplexity and the toilsome progress of attempts to penetrate into the unknown would be different from what we find them. Developed first in accordance with the needs of practical life, and in its lower forms adjusted to a narrow range of possible experiences, the unusual and baffling need not be especially complex. It need only present unfamiliar connections of things. Therefore, as we hope to show in the section devoted to experimental data, puzzles largely derive their "*quale*" from the fact that they thus set against the current of natural tendencies and habits.

II.

Puzzle Interest. Questionnaire Results.[1]

In order to secure data for the study of interest in puzzles, a syllabus was issued, containing the following questions, among others.

1. Have you ever been interested in Puzzles, some of the types of which are as follows:

A. *Mechanical Puzzles.* (1) Those dependent largely upon manual control and dexterity, as Pigs in Clover, Spider and Fly, and many others. (2) Those dependent on some trick or secret, as magic match-boxes, purses, money-banks, ball in barrel, key and ring, etc. (3) Those puzzles, as united hearts, interlaced triangles, ball and spiral, links, circles, and the like, wherein the problem is to separate the parts, or to remove rings or loops of cord from wire circles or loops of twine, etc. (4) More complex forms such as Chinese rings, Tower of Hanöi.

B. *Geometrical Puzzles.* (1) Dissected or combination puzzles, analogous to dissected map game, where geometrical forms are to

[1] For the data obtained, the writer wishes to express his thanks to a large number of persons who contributed valuable individual reports, and also to the following, who collected considerable masses of data: Miss Lillie A. Williams, State Normal School, Trenton, N. J.; Dr. Gertrude Edmund, Superintendent of the Training School, Lowell, Mass.; Miss Laura Teft, Superintendent of Kindergarten, and Prof. A. P. Wills, both of the Colorado State Normal School, Greeley, Col.; Dr. Theodate L. Smith, Mount Vernon Young Ladies' Seminary, Washington, D. C.; Principal George C. Purington, Maine State Normal, Farmington, Me.; Prof. Will S. Monroe, State Normal School, Westfield, Mass.; Prof. Noble Harter, Superintendent of Schools, Brookville, Indiana; Superintendent C. L. Hunt, Clinton, Mass.; Principal F. W. Doring, High School, Woonsocket, R. I., and many members of the Association of Collegiate Alumnæ.

be constructed from given fragments; or to cut a given figure into a certain number of pieces so as to produce another given figure, etc. (2) Other geometrical puzzles involving movement and position, as arrangement of counters or checkers in certain forms, arrangement of cards in peculiar squares or other orders in which they possess a unique sequence. (3) Tracing of intricate figures, as mazes, labyrinths. (4) Chess and checker problems. (5) Ferry-boat and railway switching problems. (6) The famous 14-15-16 puzzle, and those of similar nature. (7) Geometrical figures made with given numbers of matches, and the like.

C. *Physical Puzzles.* Where the play is upon some unusual or unexpected effect of well-known physical laws, as those of gravity, equilibrium, motion; blowing a cork into a bottle; removing a napkin from beneath a glass of water without touching or removing the glass; balancing coins, horses,—are a few of the many puzzles of this type.

D. *Arithmetical Puzzles.* Very numerous. Chiefly involve peculiar and unfamiliar relations and properties of numbers.

E. *Quibble and Catch Puzzles.* Also numerous. To give a single instance: how take one from nineteen and leave twenty?

F. *Language and Word Puzzles.* Riddles, rebuses, enigmas, charades, conundrums, anagrams, hidden words, word squares,—diamonds, etc., beheaded words, dropped letters, doublets, and many others.

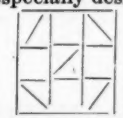
2. When was your interest in any of the above types of puzzles greatest? In what kinds did you find most pleasure? *Did the order or preference change with age, and if so, in what direction?* Please name any puzzles that have lately interested you. Will you kindly name and describe what seems to be the best puzzle you have seen?

Remember that answers in the negative are not without value.

3. When are you, or were you, most interested in working out puzzles—when alone, or during a social evening? Are they ever resorted to as a relief from ennui? Ever as a mental gymnastic, or is the competitive spirit uppermost? Do you usually persist until the problem is solved, or do you give up easily?

4. Do you know persons of pronounced puzzle interests? If so, please state age, sex, temperament; are they usually strong in school work or study? In what kinds of mental work do they seem most efficient—mathematics, physics, literature, etc.? Do they seem especially original or inventive?

6. Please note cases of children proposing more or less original riddles, even in such simple forms as "Guess what I have!" "Guess who!" where the question is clearly put as a poser, and not simply to attract attention. Also any cases of punning, making charades, guess-games, more or less spontaneously entered into by children. Any cases of "stumping" or "daring," where the task is really a puzzle and not merely a feat of strength or courage. The ages of greatest pleasure in riddles, as well as in conundrums and puns, are especially desired.



Trace a figure similar to the illustration without retracing any lines, and without lifting the pencil. Note time required for solution. How many trials before successful? Mark starting-point in each case with a large dot, and enclose the paper in your answer to this syllabus. Any notes on your method will be of value.

9. Is the puzzle-loving mind or state like that of the scientific man bent on solving problems of laboratory or study? Or do puz-

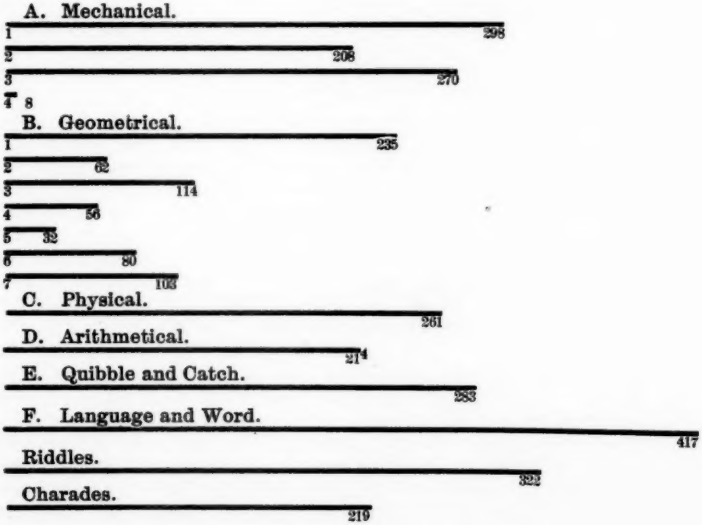
zles cultivate love of unsolvable questions, and make one impractical? Is it in danger of becoming an absorbing passion? What is the educational value of puzzles? Do children ever get nervous about difficult ones?

In response were received 556 reports, many complete, others containing replies to only a few of the questions. Of the 556 papers, 416 were from women and girls, 140 from men and boys. Ages varied from 10 to 40 years. Most of the returns, however, were from pupils in normal and training schools, with ages ranging from 16 to 30 years. Less than two per cent. report no interest in puzzles. A considerable number testify to only a mild interest.

Distribution of Interest. The chart on page 449, based upon the replies to the first question, furnishes a graphic demonstration of the wide variation of puzzle interest. Every mention of interest in any of the types is recorded. Language puzzles are more generally interesting. Next in order come mechanical, quibble and catch problems, geometrical puzzles, etc. Among the sub-classes Riddles lead, while, as A (1), mechanical puzzles dependent largely upon manual dexterity rank second. The prominence of these classes seems easy of explanation. They naturally represent the earliest and most rudimentary aspects of puzzle interest, and appeal to many in whom that interest never becomes a plant of further growth. The "mechanical" appeals probably because the problem is presented in so concrete and portable form. The definiteness of a problem may be assumed to be a condition of its attractiveness, and the delight in physical activity may be a motive. Children often value physical activity above mental activity. The delight in perception of motion also has strong influence with the same. Several children said they liked to watch the "pigs" move in "Pigs in Clover." In such puzzles the object is sometimes a drop of mercury, which has strange fascination. Had a larger number of men been represented in the above report, the relative importance of the mechanical type would doubtless have been much accentuated. As it stands, the first three sub-classes of *A* rank high. The Riddle, as indicated elsewhere, reveals a very fundamental and universal aspect of mental activity and human interest. Its very early beginning, and its perennial and permanent charm for many, render it one of the most significant of all the types of puzzles. Quibble and catch problems, while taxing and stimulating mental alertness, also often possess an element of humor of the crude sort. The prominence of many of the geometrical types seems largely due to their concreteness and to the constructive interests of the young. The earlier

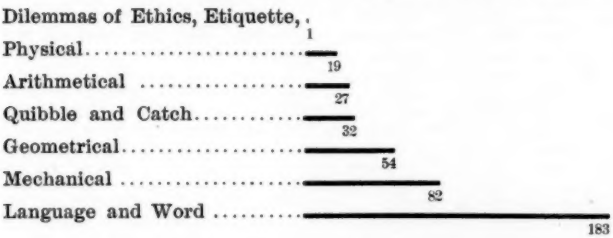
interest in geometry, as compared with number, as shown below in the curves of puzzle interest, suggests important pedagogical applications. Two classes, Logical and Philosophical, and Dilemmas of Etiquette, Ethics, etc., naturally did not find many devotees among persons of the ages of most of those reporting, although many confess to great interest in dilemmas of a personal nature, without being able to offer concrete illustrations. A few of the cases described are here given:

F., 20.—Specially fond of dilemmas of a personal nature. *F.*, 17.—Always interested in these. Remember one when quite small. Picture of man unarmed crossing a stream on a log. On his left a huge hippopotamus, on his right an alligator, behind a tiger, in front a great python. I pondered over this, trying to find some way to save him, but finally decided he must die, and thought if I were in his place I would choose the snake-bite, as that would not hurt him so much, and then I thought there was a possible chance of his jumping by the snake. *F.*, 17.—Interested in picture of ship in storm. Man overboard; large fish coming towards him. How save the man. *F.*, 20.—Very fond of scenes in books which involve dilemmas, as, where Enoch Arden returns home. In Silas Marner, etc. *F.*, 17.—Wonder what I would do if placed in an awkward position. *F.*, 18.—Interested when several persons are together to discuss them. *F.*, 18.—Very polite major bicycling with two ladies as rain began to fall. Lady Maud said, "I shall go back." Lady Mary said, "I shall go on." What did the major do?

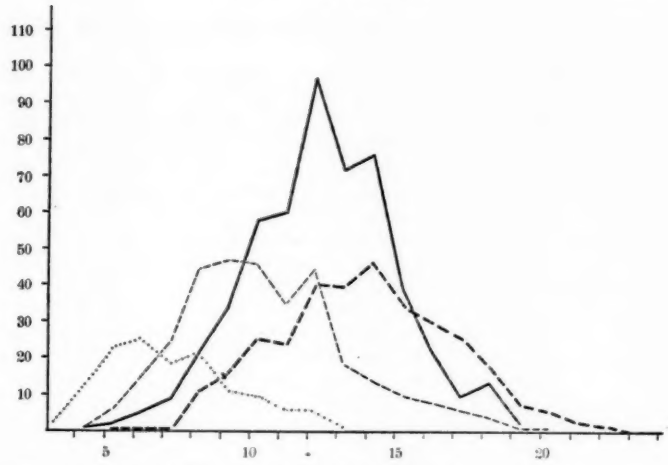


Cases of Special Interest. In examining the returns there appeared many instances of extreme pleasure in a certain kind of puzzle, an interest which, in most cases, did not later shift in full intensity to those of any other type. The appended diagram indicates the great predominance, in this respect, of language puzzles. It must be kept in mind, however, that the majority of those reporting are women. The number of persons represented is 286, some appearing more than once in the record where a new group of puzzles was taken up with the old interest.

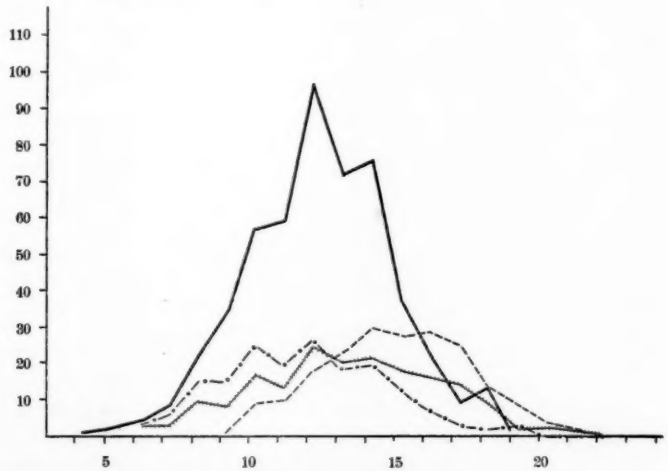
THE PUZZLES IN WHICH THE GREATEST PLEASURE IS TAKEN.



Age of Greatest Interest. The following diagrams contain not only the curve of greatest general interest, but those of the greatest interest in many of the different types. The "general" should, of course, include all the others but the incompleteness of data has prevented. For instance, some persons report only age of greatest general, while others give only age of special interest. All those, furthermore, who give but one terminus to their period of greatest interest in general or in particular, could not be included. Cases of persons of pronounced interest among those much older than any represented in the curves, suggest that the curve should not approximate so closely to the base line from 17 on. The sharp breaks are due largely to preference for even numbers in reporting. The summit of the theoretically correct curve would probably be at 11, 12 and 13. Among details of special curves, guess-games and simple riddle-making begin earliest, say, at 3 years, and reach the highest point from 5 to 8. Riddle interest proper, beginning at 4, culminates at 8, 9 and 10. Language puzzles, exclusive of riddles, are most in favor from 12 to 15, while arithmetical, beginning about 9, reach their height from 14 to 17. Geometrical, of simplest forms, similar to building-blocks and dissected games, appear at 6, culminating at 11 to 14. The curve for mechanical puzzles has a much more symmetrical form, but the cul-



Curve of General Puzzle Interest, 150 persons.
Curve of Greatest Interest in Language Puzzles, exclusive of Riddles, 78 persons.
Curve of Greatest Riddle Interest, 108 persons.
Curve of Greatest Interest in Guess Games and Original Riddles, 121 persons.



Curve of General Puzzle Interest, 150 persons.
Curve of Greatest Interest in Mechanical Puzzles, 65 persons.
Curve of Greatest Interest in Arithmetical Puzzles, 60 persons.
Curve of Greatest Interest in Geometrical Puzzles, 53 persons.

mination is reached considerably earlier, say from 10 to 13.

The above curve, of "greatest interest," manifestly can only be roughly representative of the facts. The data are not sufficiently accurate to indicate a difference between the sexes in this regard, a difference which probably exists. But allowing for obvious sources of error, the curve indicates, with a high degree of probability, that the culmination of the puzzle aspect of the mental play instinct falls in the immediate prepubertal stage of growth. It marks the close of the period just preceding adolescence. Curiously enough the culmination coincides with the period of highest "specific intensity of life"— that is, the period when "children attain and pass the flood tide of growth and of their vitality, as measured by their power to resist death."[1] "In respect to specific intensity of life, that of girls maintains a relatively high level from 9 to 12, inclusive, culminating at 11 to 13, while that of boys maintains a high level from 10 to 15, having its culmination at 12 and 13.[2] The greater surplus of energy at this period may be one reason for the greater puzzle interest. But most important is the neurological evidence. With decrease in rate of growth of bulk of the brain at about the 9th year, at which time its weight is not far short of what it is to be throughout life,[3] it is fair to assume *a priori* that the systems of cortical association fibres now begin to develop more rapidly. And, indeed, Wernicke states that at about the 12th year there is a marked increase in the medullation of these fibre-systems, which must be present before there is great activity of reason.

Experimental data concerning growth of reasoning power, though, as yet, all too meagre, furnish corroboration for the above neurological assumption. Hancock[4] finds ability to solve arithmetical problems to vary with the rate of growth. Girls show a decreased rate at 9 or 10 and 13; boys at 8 and 14. The 12th year in both sexes is therefore a period of rapid improvement. Mrs. Barnes[5] finds the power "of legitimate and imaginative inference" to have strongly developed at 12 and 13. "On the critical side the power is rare, but when present, clear and strong from the age of thirteen and upwards." Such evidence, whatever modifica-

[1] Hartwell: "Report on Physical Training," Boston, 1894, pp. 50-51.
[2] *Ibid.*, p. 51.
[3] Donaldson: "Growth of the Brain," New York, 1895, p. 104.
[4] J. A. Hancock: "Children's Ability to Reason," *Ed. Rev.*, 1896, pp. 261-268.
[5] "The Development of the Historical Sense in Children," Part I, *Studies in Education*, No. 2, p. 52.

tions later researches may impose, points to a rapid and considerable expansion of "elaborative" mental processes in this period. It is thus an intellectual age par excellence.[1]

Now, as to the general relation of puzzle activity to this prepubertal period, the following tentative explanation is offered: The culmination of boyhood and girlhood witnesses the individual comfortably adjusted to his environment. Ideals have changed only slowly. New interests have not intruded abruptly. "Being a boy" or "being a girl" has, in short, become easy, and the energies are not all taxed to maintain the equilibrium of life. Thus a mental surplus which expends itself in play. Adolescence comes with its vast enlargement of horizon. As by a leap the boy has approximated to the stature and many of the ideals of a man; the girl those of a woman. The feeling of mal-adjustment, the strain of adaptation to the new and strange demands which crowd one another, may well call out all the reserve of mental and moral strength; and in the "*Sturm und Drang*," intellectual play languishes.

Furthermore, may not this prepubertal intellectual play activity bear direct propædeutic relations to adolescence? The resulting flexibility of mind, due to the breaking up of narrow modes of thought, and the accompanying increment of gain in strength and poise of intellect and will, may help somewhat to mitigate the dangers of the "new birth." While such a suggestion is, as yet, almost wholly speculative, it seems probable that more intimate knowledge of adolescence may show that in spite of its apparent abruptness, the organism has long been gathering its forces for such a crisis.

Change of Order of Preference with Age. The obvious facts are: (1) That order does change with age. (2) The change is from the simple to the more complex forms. (3) Riddles, Rebuses and simpler Charades give way to the Conundrum, Enigma, Word Square, Hidden Words, and the like.

No typical order is derivable from the data. Wide individual differences prevail. The following order seems, however, roughly true for the largest number of persons: (a) Riddles and simpler forms of Mechanical Puzzles. (b) Charades, Conundrums, Quibble and Catch Puzzles, simpler forms of Geometrical Puzzles. (c) Arithmetical, more complex Mechanical, Geometrical, Language. (d) Logical and Philosophical. Dilemmas of personal nature.

In reading the returns it was impressive to note the logical development and the growth in mental versatility indicated

[1] Twelve is the age when Rousseau would have Emile, after the years of freedom from restraint, put under formal tuition.

in the shiftings of interest, as age increases. The Riddle, Rebus, Charade and simple Mechanical Puzzles, which, at most, call into exercise only the crudest and simplest logical methods, gradually, but perhaps not uniformly, yield to greater complication and refinement of procedure in the Word Square, and the like, with their more sustained and definite use of inductive and deductive processes. In complex Mechanical, Arithmetical and Geometrical problems, not only is procedure more involved, but the value of increased mental span, of improved geometrical imagination, is much in evidence. With age, the increase in value of "information," or knowledge, for successful solutions, notably of Language Puzzles, is also marked.

Original Riddles and Guess Games. This category belongs so largely to games, rather than to mere puzzles, that it seems best to give to it separate consideration. Here one glimpses the incipience of the originating and the inventive faculties in a somewhat striking way. The appended records furnish a better picture than would any description:

F., 9.—Little girl, has made up a half-dozen very good riddles. *F.*, 6, 8 and 13.—Very fond of making up riddles. Will sit by the hour and give riddles to each other. *F.*, 12.—Composes the names of "hidden trees" and other hidden words. *F.* and *M.*, 9.—Children of this age greatly interested in riddles. Will spend nearly all the noon-time intermission at school in telling and trying to guess riddles. Nearly all boys. *F.*, 5 and 6.—After looking at picture-book, one said, "Now let us play; you take the book, and I'll guess what picture you turn to." *F.*, 8 and 9.—In the evening the little girls would sit on the lawn and play a game, such as, "I see something beginning with O.; what is it?" When raining would play charades. *F.*, 5 and 6.—I'll guess what picture you turn to. *M.*, 6.—Very fond of having person guess any problem he may put. Gets angry if some one does not guess. When he has heard a riddle will apply the same conditions to some other object, and then is much vexed when laughed at. Instead of saying, "What is it a wagon cannot go without, and yet is not necessary to it?" W. B. might say, "What is it a lamp cannot go without?" *M.*, 8.—Likes riddles, and never so happy as when some one will sit down and amuse him with such questions. Having heard, "What has eyes and cannot see?" he made up a riddle, "What has feet and cannot walk?" Answer, a table. *M.*, 8.—Made up riddle, "Why is an apple round like an orange?" *F.*—Guess color, the initial being given, or person's name. *M.*, 4.—Hands behind back, "I've got something round and green, with a hard coat inside the green. Guess what it is?" Answer, walnut. *F.*, 6.—Often goes out of the room, wraps things up, and then will come in saying, "You can't guess what I have here." *F.*—"Guess what I have?" "Guess where I'm going?" etc., are a few of many forms of problems, mostly given, perhaps, for the sake of attracting attention, but occasionally through real desire to put a problem. Children usually prefer to set the problem for others; some, however, have a passion for answering. The passion for dramatic action also finds frequent expression in their delight in charades, many cases of which are reported. A single instance: *F.*, 17.—When

11 or 12 years old, my schoolmates and I were very fond of charades. Saturday afternoons we would arrange curtains, dress to represent different characters, and spend whole afternoons acting charades. Another reports, when 6 or 7 years old, playing charade. In one a snake was represented by the children lying on floor and crawling in imitation of snake.

Some of the Motives and Conditions favorable to Puzzle Activity. Of 419 who answered Question 3 of the syllabus, 189 prefer to be alone when working puzzles; 22 sometimes like to be alone, sometimes prefer trying in a company; 224 are most interested in puzzles during a social evening. With many, the desire to work alone is evidence of greater interest. Some say they prefer to try the more difficult ones away from the distraction of companions. The significant fact here, however, is that almost half the total number reporting prefer to try puzzles alone. This number includes nearly all those of "pronounced interest." The replies to the following questions were vague and unsatisfactory: "Are they (puzzles) ever resorted to as a relief from ennui? Ever as a mental gymnastic or is the competitive spirit uppermost?" Many seemed unable to tell why they attempted to solve puzzles. But the results, such as they are, yield the following statistics; 67 say they resort to puzzles as a relief from ennui; 47 as a mental gymnastic; while 165 find the competitive spirit to be stronger than the above motives. Many of the above 165 are persons of no great interest in puzzles. The interest in such cases is by contagion and imitation. The desire to work alone, and for the pleasure of the work, which is predominant in those who testify to a marked puzzle interest, strongly suggests some of the conditions of scientific research, which often needs to be unsocial and for the love of the work, in order to be most effective.

In response to the last question, "Do you usually persist? etc.," a large majority claim to persevere. But most important of all the facts discoverable in the responses to the question, as to motives, etc., are the persistence and tenacity with which a puzzle bids for attention, and holds it. The problem often needs to be examined in a most casual way, and for the briefest time, in order to cramp the attention and give to the relevant ideas, together with desire to solve the problem, many of the features of a fixed idea or dominant impulse. Nothing perhaps more strongly evidences the instinctive substrate of puzzle activity. The following are a few of the cases which have called attention to this point:

M., 38.—"I dislike puzzles extremely. But if I 'fool' with one for a little while, it seems to challenge me, to 'dare me' to work it, and I cannot 'let go' until it is solved. On one occasion, I was busy in conversation in a store, and my hand and eye accidentally

and automatically became engaged with the 'Tower of Hanöi.' Before I fully realized what it was all about I was deep in the attempt to solve it."

F., 17.—"I generally lose patience the first time and put the puzzle away then. I take it up again in a few days and try at different times until I get it."

F., 13.—"Have often become disgusted with puzzles and tried to push them aside, but they would remain in mind and bother me so much that I would be compelled to go and finish them."

F., 17.—"Generally work until I get the puzzle. Always feel uneasy if I don't and can't settle my mind on anything else."

F., 14.—"If it takes two or three days, I work till I get it."

F., 14.—"Usually persist until I get angry with puzzles. Then I will put them aside for awhile, but invariably take them up again."

Pascal somewhere says that men strive to accomplish difficult feats in a game, in order that they may boast of it to their friends. That the competition motive also accounts for much play activity needs no further evidence, but in view of the considerable number who solve puzzles for the mere pleasure, who may not even have in mind the gymnastic value, there is ground for the belief expressed in the opening chapter of this study, that puzzle activity is an expression of an intellectual play instinct. Any number and all varieties of conscious motives may cluster about the activity, but the instinct furnishes the initiative, and brings it about that puzzles are a congenial exercise.

Opinion Concerning Educational Value. In answer to the question, "What is the educational value of puzzles"? a wide variety of responses was received. Nearly all, however, ascribed greater or less value to puzzle activity. A few considered it harmful or useless. The pooling of opinion results as follows: Heading the list of those capacities which are thought to be improved by such exercise, stands reasoning or thinking. 37 persons say puzzles make one a "good thinker"; 72 persons say it cultivates "reasoning faculty"; 18 emphasize "accuracy of thinking"; 19, "trains one to think quickly"; 31 specially urge that it develops "tenacity, perseverance, patience"—as one writer put it, "it gives a moral training"; 35 mention its value in developing "power of concentration"; to 25 it has great value as "general mental discipline"; 19 mention that it trains the "power of close observation and accurate perception"; 13 mention it as a valuable "memory training."

Although such a consensus can have only suggestive value, it must be said that many of the above opinions are from teachers of long experience, many of whom write from personal observation.

The almost unlimited range of puzzle material, and what is more important, the range of capacities which may through

them receive valuable exercise, reveal the thoroughness with which the intellectual play impulse is equipped to do its work in producing versatility. While most puzzles represent odd and unusual modes of presenting a difficulty, they nevertheless deal with the fundamental principles of the sciences from which their material comes. A well arranged series of arithmetical puzzles would induct one into an intimate knowledge of the more important properties of number. Geometry might well be largely appropriated in the same way. Moreover, the forms of illusion and distraction which make the puzzle quality are such as one must experience in ordinary dealings with the new. Puzzles tend to emphasize the usual sources of error into which the mind is prone to fall in its every-day experiences. In such facts as the above rests security from wasteful use of energy. So impressed with the value of the puzzle was Chalotois, a French royal commissioner of education, that in his "*Essai d'éducation nationale*," published 1763, he wrote as follows: "I take it for granted that a child already knows how to read and write, and even to draw; these are necessary. I think that the first things which should occupy him from 5 or 6 to the 10th year are history, geography, natural history, *physical and mathematical recreations*,—knowledge which is within his reach because it falls under the senses, because it is the most agreeable and consequently the most proper to occupy the child."

Cases of Pronounced Puzzle Interest. Had the correspondents, who themselves possessed pronounced puzzle interest, added their own cases to those reported under this head, the number would have been greatly augmented, perhaps more than doubled. As it stands, 128 such persons are reported, of which number 36 are women and 92 men. Distribution as regards age shows 97 to be 24 years or less; 24 from 24 to 48, and 23 from 50 to 80 years. 90 of 109 are reported to have been studious and proficient in school work. Some of the remaining 19 were reported as bright, but not interested in school work, while a few were positively dull. One city school principal writes that the majority of those in one of his classes, who solved a given puzzle most readily, were among the weakest in school work.

Among the subjects in which persons of pronounced puzzle interest are said to be most efficient, Mathematics is mentioned in 62 cases; Literature, 20; Physics, 17; Mechanics, 5; Drawing, Language, Music and History, each 3. Some, of course, excel in more than one branch. As to originality and inventiveness, 82 are reported as one or both, while 18 are neither inventive nor original. The words are

evidently used in a somewhat ambiguous sense, occasional returns showing that originality may consist in the simple ability of dazzling others with novelties, in the way of tricks, etc. Nevertheless, considerably more than half the whole number can be safely credited with real originality, though in varying degrees. Some have done creditable literary work; others are successful inventors. The temperament in only 32 cases is reported, but even these data are sufficient to prove that not only the nervous and sanguine, but also the bilious and phlegmatic may possess remarkable interest in puzzles. Below are some of the cases:

M., 10.—Bright in mathematics, but poor speller. Constantly working at puzzles, riddles and rhymes. Very original and inventive. *M.*, 12.—Very fond of puzzles. Will not give up until he can do it. Very inquisitive. Asks the "whys and wherefores" of all machinery he sees. Inventive, but poor in school-work. *F.*, 9.—Greatly interested in puzzles of all kinds; sometimes makes puzzles out of picture cards, etc. *M.*, 10.—Originated some kind of puzzle with strings—a very good one, too. When younger was very fond of guess games. *M.*, 12.—Tries for puzzle prizes. Very persevering temperament. Fond of arithmetic, specially original and inventive. Remarkable child. *M*, 13.—Fond of puzzles, and quite successful. Not very strong in school-work, but passionately fond of reading. Power of invention marked. *F.*, 15—Spends much time solving puzzles and making puzzles for magazines. Best in arithmetic. Very skillful at making things about house, being very original and inventive. *M.*, 15.—Often made puzzles for his own use. Has a book of puzzles. If he sees a good one, he will cut one out of wood for himself. More interested in physics than in any other study. *F.*, 16.—Long an invalid. During this period delighted in puzzles, original. *M.*, 15.—Fine at anything mechanical, yet not a particularly brilliant scholar. Not specially original in thought, but extremely so in practical affairs and construction. *M.*, *M.*, *F.*, 17-21.—One family. Girl almost entirely deaf and has been nearly blind. Not good in school-work; exceptionally dull at all kinds of work. Takes to music pretty well, but that is all. Poor readers, but enjoy games, riddles and puzzles. Not original. *M.*, 18.—Very queer, not very bright nor studious, but a good typesetter. Always trying something new, but seldom succeeding. *M.*, 14-18.—Peculiar temperament, no business ability, no interest in school, but great reader; best in literature; neither original nor inventive. *F.*, 10.—Would give herself up completely to puzzles, until they were taken away. *M.*, 14.—Would work for hours on a puzzle, but could not put his mind to the most simple problem in arithmetic for five minutes. Actually stupid at some things. Far behind class of pupils of his own age. *M.*, Adult—Fond of mathematical puzzles, strong in mathematics, and has done some very creditable work in literature. Not specially original or inventive. *M.*, 60.—Scholar. Especially original thinker. Never gives up a puzzle when once begun. Daughter inherits his love of puzzles and works at them with some energy. *M.*, 36.—Always prided himself on his puzzle collection and on his ability at chess. Good in arithmetic and a great reader, but seems weak-minded or crazy on mining. Not the least original, inventive or

practical. *M.*, 50.—Believe he would neglect his meals any time to work out a puzzle. Very intellectual, shrewd in business, fond of study. Specially original. *F.*, 30.—Invalid. Unusually strong mind. Fine student. Omnivorous reader. Comes of a family of inventors, and seems to inherit some of it. *M.*, 25.—As a boy I have often gone to his house on rainy days, and found him making and solving puzzles of string and wire. No interest in school-work. Specially original and inventive. *M.*, 29.—Extremely fond of puzzles. Most efficient in reasoning logically and thoroughly. Extremely original and inventive. Spirit of investigation wonderful. Devoted to experimentation and devotes most of his spare energy to it. *M.*, 52.—Minister. Especially strong in literature. More original than inventive. *M.*, 22.—Whole family very quick at puzzles. Excellent in drawing and mathematics. Is now a draughtsman. Has devised and patented several puzzles. *F.*, 17.—Whole family exceedingly interested in puzzles. Enjoying making and inventing puzzles. Every new visit to their home reveals some new puzzle which they want you to try. *M.*, 22.—Has a perfect mania for chess problems. I have seen him go off by himself at a picnic to work such problems. Is a professor of mathematics. Do not think he is specially original or inventive. *M.*, 28.—Cannot read at all. Solves puzzles of all kinds and seldom fails. Specially original and inventive. *M.*, 50.—Tries each new puzzle as soon as it comes out. Very bright, well-read man.

F., 70—It has been a serious question whether I have not foolishly spent a great many precious hours in the last sixty years which might have been more profitably employed than in solving charades, anagrams, enigmas, etc.

In a way it may have had an educational value, but that was not my *object* in doing it. It has been for *pure enjoyment, recreation* and FUN, and I have got lots of each of them out of it. I *do* think it quickens the perceptions and keeps the mind active. It has led me to study in the direction of history, biography, literature, mathematics, astronomy, and mythology, not for the sake of the *information*, but to find solution of a puzzle. I cannot remember the time when not interested in these things, and interest increases rather than diminishes with years. I best enjoy solving *alone*, as I want no hint of solution and want all the *time* necessary. Numerical enigmas interest me very much. I have learned more of Bible geography, biography, feats of valor, words of wisdom, numbers, as "enigmas of forties or of sevens," from this kind of enigmas than I ever did from consecutive reading. Very few names or places in the Bible seem unfamiliar. For years a column in —— was devoted to Biblical enigmas, which I never failed to study out, some having as many as 150 or 200 letters, and the whole being some passage or passages of Scripture.

In later years, have enjoyed anagrams very much; only last month sent in solution for a prize; conundrums and puns do not appeal so strongly to me; mechanical puzzles I like, but they tire me; I get nervous if I cannot see through them. Only yesterday I came across my "Pigs in Clover," and stopped all work till I had them in the pen. I never stop at a railway station where there are handbills, without taking some prominent word and trying how many words I can make from it. I can get few persons interested in this; have one brother and a niece who enjoy it as much as I; another brother cares nothing about it; not inherited, this interest; don't like studies requiring memory chiefly, but like anything I have to *dig* out.

M., 45.—Nervous; mechanical genius, an inventor with notions and hobbies. *M.*, 44.—Nervous; inventor; they say he has never failed to work any puzzle; designer, draughtsman, inventor and practical mechanic. The puzzle characteristic is peculiar to his family, and they all have mechanical skill, ingenuity, but little business ability. I am convinced that "puzzle" ability goes with inventiveness, mechanical skill, etc. Men excel; they excel in mathematics, physics, etc.

In a sketch of the late Prof. Sylvester, the great English mathematician, published in a recent issue of the New York *Post*, the following statement appears: "Prof. Sylvester was all his life long, down to his latest years, an indefatigable solver of exercise problems, such as are proposed in the *Educational Times*. He considered them an indispensable whetstone of the wit; his whole style of analysis carries the marks of such exercise."

The significant points, in résumé, are that the puzzle interest may endure throughout life; that, on the whole, persons of pronounced puzzle interest make a favorable showing intellectually. Most are interested in every-day problems, and succeed in practical affairs; they certainly stand high for originality and inventiveness. Is this sustained puzzle interest different in kind from that which marks the prepubertal period? With the insufficient data at hand, no satisfactory answer can be made. With some individuals, it may well be an abnormal persistence of tendencies which should have fallen away long before the individual had reached maturity. But in others, as in Prof. Sylvester, the play instinct has been fostered, apparently because it served the purpose of intellectual gymnastics. Among very old people of leisure, it may be a veritable recrudescence of play activity.

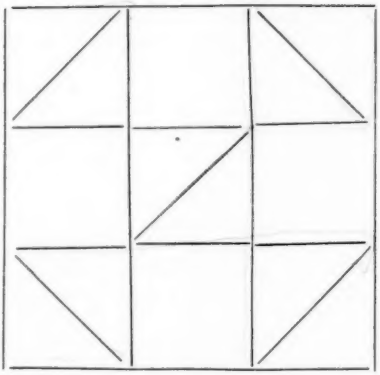
Summary.—The most salient facts concerning puzzle interest are as follows: The interest is fairly general among young people; interest is distributed over a wide range of puzzle materials. Of these, the mechanical and language groups are most popular; puzzle interest shifts with age. In general the change is from Riddles, and the simple Mechanical and Geometrical Puzzles, to more complex forms of the Language, Mechanical, Geometrical, and also Arithmetical types; the age of greatest general interest in puzzles is about 12. No difference in this respect as regards sex is discoverable. This prepubertal stage is probably marked by rapid development of the elaborative mental processes, and may be denominated as largely an intellectual epoch; puzzle interest may persist throughout life; most persons of pronounced puzzle interest seem original and inventive, and are not as a rule unpractical.

III.

EXPERIMENTAL.

For the purpose of making experimental determination not only of methods of solving puzzles, but chiefly of the development of mental adaptation in general, the following problem was submitted to children in the schools of Worcester.[1] These results were, in turn, compared with those gained from tests upon adults.[2]

The puzzle used belongs to the unicursal or labyrinth types and was devised by Tait. An illustration is here appended.



This puzzle was chosen mainly, because: (1) The problem is sufficiently difficult of solution to require a number of trials. (2) The instructions as to procedure are easily followed. The subject is required merely to trace all the lines without lifting the pencil and without any retracing of lines. A sheet containing twelve reproductions of the

[1] These tests were rendered possible through the courtesy of Superintendent Carroll and the school authorities of Worcester. I was assisted in the supervision of the experimentation by my wife, and by Mr. E. W. Bohannon of the University. The work was also greatly facilitated by the cordial coöperation of the principals and teachers of the schools above specified. The generous and able assistance of Miss Emily Viets in a series of valuable preliminary tests in the kindergarten of the Salisbury street school deserves special mention.

[2] The experiments outside the University were conducted at Providence by Dr. Hattie E. Hunt, of the Training School; and at Indiana University by my friend and honored teacher, Dr. W. L. Bryan. Their important contributions are gratefully acknowledged.

design, eleven of which were in broken lines, the other serving as a model, was supplied. When the subject had failed on one figure, he was required to try the next, and so on, taking care to mark the starting-point each time in some distinguishable way.

The figure is so constructed that the place of beginning is of prime importance. There are only two points from which successful movements may start, and these are on the inside of the figure, at the termini of the middle diagonal.

The mathematician Euler developed a formula for such figures, and for one conversant with his rules, this problem offers no real difficulty. Using such a mathematical analysis as a norm, it may be assumed, a few chance successes apart, that all cases of failure are due to an inadequate conception of the problem. So it might be possible to arrange in a series all analyses of the design, according to the degree of adequateness of conception. So much for the purpose and import of the problem.

After a series of preliminary tests upon members of Clark University, the experiment was made in the third, fifth and eighth grades of the Oxford street and Elizabeth street schools, and in the third, fifth and ninth of the Woodland street school. A later series of individual tests was made at the Salisbury street school, also on teachers of the schools of Providence, and lastly on advanced students in psychology at Indiana University. These individual experiments will receive separate consideration.

Mass Tests.

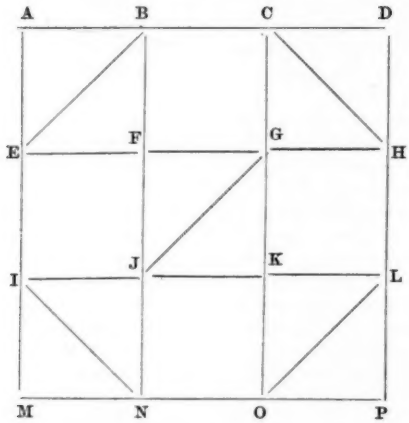
At the first three of the above-named schools, the problem was given to all the children in a room at the same time. They were furnished with uniform papers, were given explicit directions, and were more than once reminded that they could begin anywhere they chose. Twenty minutes was the allowed time. If a pupil thought he had succeeded he made the fact known, and the time required for solution was recorded. The total number tested in these mass trials was 471, distributed as follows: 169 in Grade III, 154 in V, 61 in VIII, 87 in IX.

Almost without exception the children entered into the problem with great interest, and there is every reason to believe that most of them made every effort to solve the puzzle. Participation in the test was entirely voluntary in every case. Most of the children voted that they liked puzzles, and many wished to continue after the expiration of the allotted time. There were some failures to follow the instructions, not only in the lower grades, but sometimes in the higher. In the

first series, in Grade III, Oxford street, the children were allowed to decide for themselves when they had solved the problem. On inspection many had omitted lines, and some were greatly surprised when the fact was pointed out. Others plainly had lifted the pencil in order to escape failure, while still others had retraced certain lines. As most of these mistakes seem to have been made without intent to deceive, observations were made on further cases of like kind, and will receive consideration below. After this experience the instructions were made still more explicit, with the result that the proportion of errors of this nature in later tests was materially reduced.

Results. From these mass tests only qualitative results of the more general kind were expected. The average number of trials does not greatly vary with age, although the younger children show a slightly greater number. The circumspection with which pupils proceeded increased noticeably up the grades. The youngest children worked rapidly, and failing, passed quickly to the next figure. While this was also true of some in the higher grades, it was nevertheless patent that these pupils exercised more deliberation, and there was a greater attempt to analyze the figure and to determine the causes of failure. While it is impossible to make a precise quantitative statement of the increase in *variety of attack* in the higher grades, this fact was most prominent. The younger children, with few exceptions, show little ability to profit by their errors. They either began in the same place in many successive trials, and seem to have repeated their former unsuccessful movements again and again, or their variation was too slight to be of consequence. In Grade V there is a smaller proportion of these automatic repetitions, while in VIII and IX they had still further decreased, although it must be noted that such cases occur even among adult subjects. As regards ability to profit by errors: In case of a few pupils of Grade III, and in still larger proportion with those of V, more marked variations are produced by a former error. The new movement shows vague appreciation of the total conditions of the problem, and there is less focusing on single lines than calculation of the effect of moves on several lines. In VIII many show radical reconstruction of order of moves, following failure, and usually the *adjustment is prompt.* In V radical reconstruction does occasionally occur, but more slowly. It appears that among younger pupils, error must be repeated before its full significance is appreciated. With age, the movements seem more and more directed by deliberation and fuller realization of the conditions of the problem.

Conventional Reactions:—Examination of the nature of the beginnings in the separate trials brought forward some facts of interest. All subjects were explicitly told to begin wherever they chose, and among the younger children the reminder was repeatedly given. In spite of this, however, the beginnings are of a very conventional sort.



In the figure here given the junctions of the lines are lettered, for convenience in indicating various parts of the figure. Barring a few cases, where beginnings were made in the middle of some line, these junctions represent the range of starting-points. If the "dice were not loaded," the starting-points would be distributed equally among 16 positions. As there are 4 inside and 12 outside points, 75% of the beginnings, on such an assumption, should be on the outside.

Results:—The percentage is too great in favor of the outside among the younger children, and too small among the older. In Grade III, 87% of *all beginnings* are on the outside; in V, 83%; VIII, 73%; while in IX only 68%. For adults only 61%. In only a relatively small number of cases is the starting-point of the first trial on the inside. A view of the distribution of beginnings among the various outside points throws yet more light on the question. As to beginnings in upper left-hand corner: In III, 41% of all beginnings were at *A*: V, 29%; VIII, 22%; IX, 15%; adults, 10%. Furthermore 57% of all beginnings are somewhere on the top

tine *(A, B, C or D)*, while only 10% were on the bottom line *(M, N, O, P)*.

Now, if we consider the *number of individuals* who begin at any given point, we find that 57% of all persons begin sometime at *A*. In Grade III, 66%; V, 66%; VIII, 45%; IX, 43%; adults, 36%. When the construction of the puzzle is recalled, and especially the fact that only beginnings at two inside points can possibly lead to success, this predominance of outside starting-points, notably among the younger children, is of considerable significance. It shows that children are probably under the sway of a deeply set habit, learned from reading, writing and drawing. This leads them to begin at the top and generally at the upper left-hand corner of the design. Failure is not effective in inhibiting this tendency. In older pupils and in adults this tendency seems, in part, overcome by other conditions, although it still exists. Their range of experience is wider, and hence the predominance of any one fixed tendency is minimized.

Furthermore, these beginnings throw some light upon the perception of the figure. Very many of the youngest children invariably went round the square, suggesting that this form, which is at once most familiar, and so placed in the design as to constitute the outline of the whole, catches the attention and holds it at the expense of the other possible combinations of form. This view is corroborated by the testimony of adults, namely, that first trials were often haphazard, preceded by little analysis of the figure. This, together with the general absence of deliberation in children, would seem to prove, pretty conclusively, that nearly all first trials, at least, are not planned, and hence the outline of the figure, and the usual method of drawing, are all the more potent in determining procedure. Still another fact points to the rôle of these habits. Many who studied the figure discovered the error of beginning on the outside, by their inability to dispose of the inner diagonal. This usually suggested that the diagonal be disposed of first, and hence beginnings on the inside were almost imperatively indicated. In spite of this difficulty, however, one which nearly every subject must have experienced in some degree, the outside initiative still tended to persist.

Individual Tests.

In order to observe more exactly how far age influences capacity to vary effectively, to profit by errors, a series of individual tests was made on pupils of the Salisbury street school. Of a total of 64, 11 boys, 10 girls were from Grade III; 11 boys, 12 girls were from V; 11 boys, 9 girls from

VIII. Both boys and girls were chosen in alphabetical order. They were brought into a room in groups of six, and two children were assigned to each of the three observers. They were given instructions as above described, and the observers attempted to record each move made in the attempted solution. While it would be highly desirable to make a quantitative formulation of results, the complicated conditions prevent. So many are the possible permutations, so diverse may be the causes of any slight change, and so unequally effective in altering the picture of the whole is such a variation, depending on whether the change comes near the beginning or toward the close of the trial, that in view of all these difficulties, resort must behad to the clinical method of presenting typical cases. The following seem as nearly representative, as possible, of their respective grades. In the tables a sufficient part of the total movement in each case is given to convey a fairly accurate notion of the whole. The letters refer, of course, to the lettered diagram above :

ORDER OF MOVES IN TYPICAL ATTEMPTS AT SOLUTION.

GRADE III.

R. F., boy, age 8.
M I E A B C D H L P O N M
C H L O — N I — E A *
E A — C H*
E B C H L O N I E
E B C H L O N I E
E B C H L O N I E
I E F G H L P O N M I†
E F G H D C B A E I M N O P L H C G K O
M I E A B C D H L P O N M
M I E A B C D H L P O N M
M I E A B C D H L K J
E B C H
M I E B D H L P
* Omitted certain lines. Put in only the diagonals.
† Retraced from I to A a second time.

B. G., boy, age 7.
A B C D H L P O N M I E A
A B C D H L P O N M I E A
A B E I N O L H C D H L P
A E I N J F E B
A B E I M N I J K G J F B C G H C D H L P O L K O N J
A B E I N O L H C B I J N +
I N J G F J N I E B A E F B C D H C G
A B E F B C D H C G F J N M I N O K J G H L O P L K G
E I N J G C B A E B F G I M N O K +
A B F E I M N I J F G C B

G. A., girl, age 7.
$\frac{BC}{2}$ C D H L P O N M I E A
$\frac{BC}{2}$ C D H L P O N M I E A
$\frac{BC}{2}$ C D H L P O N M I E A

C D H L P O +*
B E F G H +
C D H L P O N M I E A B C G K O L K J I N J G E B F
C D H L P O N M I E A B C H G F B E F J G K J N I J
B F J G C D H – – – – – – – – – – P O N J I M I E +
G J F B E A B C G H C D H G K G F E I J N +†
C D H C B A E B F G J I E F G H P
C D +
G H D C H L P O L K G C B A E B F +

* Gets inside for first time.

† Said she would begin on inside; perhaps she could do it that way.

J. F., girl, age 8.
A B C D H L P O K J I M +
A B E A B F J +
A B C D H L P O N M I E A
E B F J N I J G H C D H L K O L P O N M +
A B C H G K J G F E B F J I N M +
H +
J G C H G F B E I M N O L K J I N J F E A D H L P O K G

GRADE V.

M. C., boy, age 10.
C H G C D H L P O L K G F B E A B C
I J K L H D C B A E B F G C H G J F E +
J G K J +
J G C H D C B A E B F +
Never tried a puzzle before. Cannot always repeat his success.

J. D., boy, age 11.
P O N M I E B A E
D C G H C
M I N
A B E B C G
P O L K O N J G
P L O N J G H
D C B A E I M N O P K O L
F E A B
B A E F B C I J F G J K G H L K O L P O N M I N J

G. W., girl, age 11.
A E I M N O P L H D C B A
E A B F E B C G F J +
A E I M N I J N O P
B A E F G H D C B F J N +
B A E B F J N O +
B A E I M N O P L K J I N J +
C B A E I M N J +
D H L P O K G C H G F E B
G J N M I E F J I N O K G F B A E B C G H C D +
G J N M +
G J N I J K +
G J +

J. M., girl, age 10.
A B C D P O N M I E A
E B F J I N O L K J G F
C H G J I E A B C D H L O P L
O L K G J N O P L H G F E I
N I M N O P +
J G H L K J F E B A E I M N I J N O L P O G F +

GRADE VIII.

F. E., boy, age 13.
N I M O P L O K +
B E A B C D H C G +
E B A E I M N O P L H D C B F +
H G F +
D H L P O N M I N J +

E F B E A B C H
N M I J K G J F G +
I J G C D H +
Traces before marking.

M. E., boy, age 14. D C H G C B F E B A E I J F G J K G
C B F E B A E I J +
G C H D C B F E B A E I J F G K O L P O N M I N J +
Studies before marking.

E. A., girl, age 12. M N J F B E
N +
H D C H G C B A E B F E I M N I J N O P L O K J
J E K O P L O N J I N M I E F B E A B C D H C G H L K J F G
Studies it first, tracing with finger. Sees quite far ahead that she cannot succeed.

L. D., girl, age 13. M N O P L +
A E I M N I J F E B F G +
B E F B C G H C D H L P O L K O N J K G F J I M N I E A B
H C D H G C B +
E I M N I J N O K L O P L H G K J G F E A B F J
G J I E A B F E B C G F J N M I N O P L O K L H D C H G K J

Even from the cases given, one can scarcely get a full realization of the remarkable increase in variation from III to VIII. As in the mass tests, while automatic repetition, or slight and inconsequential variation, is the rule in the lowest grade, in V there is already appreciable growth in resource, initiative, intelligent utilization of previous failures. In VIII the progress has been relatively greater still. Even in the higher grades, however, appeared a few cases where routine prevailed. In other instances, where variation was considerable and intelligent, there were occasional relapses into routine. This latter may, in part, be due to a temporary relaxation of attention.

On a basis of the study of the records, supplemented by observation as well as occasional questioning of children in III, the following analyses are offered as fairly descriptive of their procedure; they throw much light on the above mentioned characteristics of their movements. There are two slightly different cases: (1) The child discovers in a just completed trial that one or more lines are omitted. These lines sometimes, of course, are on the inside, sometimes on the outside of the figure. No matter. The child starts again at the old beginning place, or as near to it as possible, and modifies his former movements just enough to include the line or lines in question. This generally results in the omission of other lines; and this, in turn, leads to similar slight modifications, and so on. (2) Lack of foresight is manifested in a slightly different way. Again repeating his former movement, the pupil comes, say, within a single move of his old dilemma. Only then does he veer, and then only just enough to avoid the immediate pitfall, without regard to

others into which his change hurries him. Comparing the most successful adaptations of these children with those of higher grades and of adults, the following seems in general true: The young children succeed through a long series of slight variations. Occasional relapses into movements which are useless, indeed occur; but the trend is by a slow and primitive method of exclusion, toward the goal. Among older persons, the adaptation is usually approximated more by leaps and bounds, that is, through wider variations.

Résumé. Without attempting here to coördinate intimately the main results of the above experiments, some such statements as these seem warranted: The lack of circumspection, the conventional beginnings, the automatic repetitions of former movements, the slight and inconsequential variations, the frequent relapses into former routine, after failure of a slight variation, in short, the general tardiness in profiting by errors, of children in III, slowly makes way in older children for greater prevision, more adequate analysis of the design, less conventionality and automatism in procedure, more radical reconstruction of plan in successive trials, all of which leads to greater promptness in profiting by mistakes.

Experiments upon Youths and Adults.

A large number of older persons, perhaps 300 in all, described their method of solving the same puzzle. Of this number, 72, who had some training in introspection, were given the task as a special problem. When they had successfully traced the figure, some such questions as the following were asked: 1. Why did you begin where you did in each trial? 2. Did you study the figure before beginning? Before each trial? About how long in each case? 3. Into what did you resolve the figure? Did conception of the figure change with successive trials? 4. Do you usually plan work thoroughly before beginning? How is it in composition, etc.? Some typical replies are here arranged so as to furnish a picture of the several degrees or levels of intelligent procedure, and to indicate some of the relations of visual and motor-mindedness.

Simpler modes of procedure: *F.*—No reason in beginning where I did in No. 1. Later attempted to cover lines not covered before; studied figure somewhat; 11 trials, 18 minutes. *M.*, 24.—Studied 20 seconds; began on outside; tried several times from about the same point; then began in the middle; chiefly guess. *F.*—No reason in No. 1; did not study; generally start in quickly on any subject. *F.*—No reason; changed beginning point because I could not cover lines first time; studied very little; did not analyze; 7 trials. *F.*—No reason; changed because I could not cover all lines; studied figure somewhat, but did not analyze; 5 trials. *F.*—No

reason; in second or third, saw that I could cross from one square to another by use of diagonals, and then fill out square; studied very little; generally start right in, then see my mistakes and correct them. *F.*—No reason; no reasons in others, but began at just any point. *F.*, 19.—Started in upper left-hand corner simply from force of habit; in fifth trial started at middle diagonal, because that had given trouble.

The following seem to represent some advance over the preceding: *F.*—No reason in No. 1; then the figure reminded me of another puzzle, but my trial did not succeed; in No. 3, after starting, saw my way; studied a little, then would try; saw figure first as a cross, then at last as squares and diagonals. *F.*—"E" looked like a good beginning; in No. 2 had another plan; in No. 3 supposed should have to begin somewhere in the middle; studied figure some time; generally studied first and arranged later; saw the figure as a lot of squares with diagonals. *F.*—Started to draw the square first, but at the same time wanted to get in the diagonals, so began at E. M.—No. 2 had no particular reason for beginning at N; in third, started with J G because it had given trouble; did not study figure much; usually do, however.

Some of the best types are as follows: *M.*, 29.—Mathematician; began at middle diagonal; sought points possessed of more than four lines; that suggested inner diagonal. *M.*, 33.—College professor; first tried by horizontal sections; then sought some principle; middle diagonal caught my attention; it must have some significance; after long inspection began there, and succeeded; in working puzzles always found it necessary to seek for a principle, rarely getting anything by chance or luck. *M.*—Studied short time; saw I could finish inner square first, then complete the rest. *M.*, 31.—Have had experience in mathematical puzzles; spent four minutes studying first figure before starting; about three minutes for second, which gave correct result; found myself continually running ahead of the lines to see that I kept out of the tendency to close myself in.

Visual and Motor Types: Although many of the cases already given suggest the somewhat fundamental differences in procedure of the eye-minded as compared with the motor-minded, this distinction seems sufficiently important to warrant special attention. The following cases are in point: *M.*, 60.—Worked it out by eye before using pencil; noticed continuity of  and  ; saw these two could be easily associated; then tried the inner oblique line, first making sure of connections of terminus in right-hand lower corner. *M.*, 26.—Tried to imagine the figure as two, or at most, three definite figures as follows:  ; saw I could trace the first two continuously, but did not know how to dispose of the diagonal in the middle; then I began disposing of it first, and the solution came with surprising quickness. *M.*, 23.—Looked it over before using pencil; two trials; in composition always plan arrangement or treatment before using pen.

While it is more difficult to point out those of more pronounced *motor-type*, a few, such as the following, seem to be dependent almost wholly for a connected view of the conditions, upon first tracing the figure. *M.*, 20.—Began on outside; rather haphazard beginning; repeat same beginning in order to hold it in mind; memory of figure improves with trials. *F.*—Did not study figure; my first

failure gave clue to second figure; generally start in upon work without thinking much about it; 4 trials. *F.*, haphazard beginnings; generally start right in, then see my mistakes and correct them; 3 trials. *F.*, nearly all early beginnings haphazard; usually start out at once, and see what I can make of it, and then I study it; 8 trials.

With persons of motor-type, there is a more or less tentative marking at first; the mind hovers until the actual experiences give a clear-cut notion of all the conditions of the problem; many of the early trials often seem aimless, but the record shows that although the "*motiles*" make a larger number of trials than the "*visives*," they succeed about as quickly. The "eye-minded" make few trials, and each stroke of the pencil is of high purposive import; they do their "fumbling" mentally; that most persons, however, are not predominantly eye or motor-minded, but rather belong to the "mixed type," goes without saying; the majority of those reporting are probably of the latter.

Abstracting for the moment, from the distinction just made, and considering the individual cases above given from the standpoint of the degree of intelligence of procedure indicated, it is possible to discriminate at least two, and perhaps three "levels." (1) *Conceptual.*—All those cases in which the search for a principle dominates, where careful analysis of the problem, where comprehensive appreciation of relations are marked characteristics, belong in this class. The word "Concept" connotes this level, which, however, is marked off from lower orders by no fixed boundaries. (2) *Receptual.*—Most of the numerous cases which fall below the highly rational types above designated belong here. These cases present almost infinite variety and resist hard and fast classification. They of course include eye-minded, motor-minded and the mixed type, just as does the conceptual group. They also represent varying degrees of approximation to the Conceptual. The word Recept designates the mental processes involved rather than Concept. The word is used with the realization that it connotes a lower stage of mental process than often appears in this group. But with this qualification it has value, in lieu of a better, in such a crude grouping. Reason, acute analysis, is here supplanted by a more hazy and insufficient realization of the relations involved. Errors, only by a considerable repetition, are able to produce any marked change in the order of the move; one sub-class may be distinguished, however, and some of its characteristics may be held to represent fairly the whole group. This subsidiary group we venture to designate "*Receptual motor.*" It possesses the following characteristics: There is a very great dependence upon preliminary tracings, for a realization of the conditions of the problem. Unlike the Conceptual-*motiles*, however, the experience gained does not result in a comprehensive concept of the

problem; there is no prompt discovery of the principle involved. With some, a Micawber-like disposition to wait for the difficulty to solve itself, seems prominent; some cases reported show as many as 15 trials, all in a measure haphazard; in others, the right notion of the figure slowly dawns after, say, 8 or more attempts. Repeated failures by a sort of summation thrust their import upon the subject, and like a composite photograph bring out the essential features of the required complex. While there may be considerable appreciation of relations, either the ability or the disposition to reason out the problem promptly seems lacking. This group, however, ranks higher than that of "sense-trial and error," to be considered later. Manifestly, it would be far from the truth to assert that a given individual in the prosecution of every sort of task need exhibit the same "level" of method. In proportion to the unfamiliarity and in general, the difficulty of the problem, his procedure may descend almost or quite to the lowest "levels." This fact must not be forgotten. Probably all inventors and discoverers often exhibit traces of many "levels" of procedure in the course of a single adaptation.

Of great value for the psychology of scientific method would be the detailed account of the procedure of the most successful experimenters. Unfortunately few have left even a general record of the kind in question. But there are enough data at hand to show, at least, how erroneous is the popular notion that the great discoverer in science somehow marches straight to the truth by some divine unerring method. "In all probability," says Jevons, "the errors of the great mind exceed in number those of the less vigorous one. Fertility of imagination and abundance of guesses at truth are among the first requisites of discovery."[1] Faraday said: "The world little knows how many of the thoughts and theories which have passed through the mind of a scientific investigator have been crushed in silence and secrecy by his own severe criticism and adverse examination; that in the most successful instances not a tenth of the suggestions, the hopes, the wishes, the preliminary conclusions have been realized.[2] Mach[3] and others have shown that accident may play an important rôle in discovery, but accidents can only be utilized by a mind quick to appreciate their significance. Newton seems to have been at times able to establish a theory by a series of experiments so carefully planned that

[1] "Principles of Science," p. 577.
[2] Quoted in "Principles of Science," p. 578.
[3] "On the Part Played by Accident in Invention and Discovery," *Monist*, Jan., 1896.

few, if any, were abortive and almost every one crucial. But of course we cannot tell how much preliminary mental "fumbling" took place in order that the happy result might be accomplished. Edison's account of the invention of his electric light gives an insight of rare value into methods of invention. "Through all those years of experimentation and research, I never once made a discovery. All my work was deductive, and the results I achieved were those of invention pure and simple. I would construct a theory and work on its lines until I found it was untenable. Then it would be discarded at once and another theory evolved. This was the only possible way for me to work out the problem. . . . . I speak without exaggeration when I say that I have constructed 3000 different theories in connection with the electric light, each one of them reasonable and apparently likely to be true. Yet only in two cases did my experiments prove the truth of my theory. My chief difficulty was in constructing the carbon filament. . . . . Every quarter of the globe was ransacked by my agents, and all sorts of the queerest materials used, until finally the shred of bamboo, now utilized by us, was settled upon."[1]

This method of Edison certainly does not belong to the highest level, but for exhaustiveness and for the particular demand of his problem, who can say it was not the very fittest? These data are not sufficiently specific to throw much light on our special problem, but they suggest the possible variation of method, with the task in hand, and also indicate somewhat of the importance to psychology of a mass of more specific data concerning invention and investigation.

Summary.—As to the method of youths and adults: 1. Nearly all study the figure; many see several moves in advance; few successive trials are without circumspection. 2. A large majority, nevertheless, make a haphazard beginning at first. 3. There are wide individual differences in the conception of the figure; and this often changes in successive trials. 4. Most are early struck by the middle diagonal as a clue. 5. As compared with children, nearly all profit quickly by errors. 6. Two stages or "levels" of procedure and adaptation are roughly indicated: the conceptual and the receptual. 7. Eye and motor-mindedness have considerable influence upon the resulting adaptations.

Animal and Child Method.

Certain considerations demand a wider survey of the prob-

[1] "Talks with Edison," G. P. Lathrop, *Harper's*, Vol. 80, p. 425.

lem of conscious adaptation. If, as every one to-day agrees, life is "the continuous adjustment of internal relations to external relations;"[1] if the whole nervous system of organisms is a differentiation of tissues with the supreme function of preserving the results of former adjustments and the effecting of new ones; if, furthermore, for creatures in the ascending scale of life, consciousness is increasingly the instrument or the concomitant of all adaptations to new conditions,—then may we confidently seek a genetic view of the natural forms of adaptation, of the natural logic which organisms employ in dealing with novel situations. Of manifest importance to biology and psychology would be the natural history of such processes, from the lowest forms of conscious life to man, as well as from primitive man and the child to the adult scientest. Some studies of animal method, notably those of Romanes, Lubbock, Lloyd Morgan, Binet and Hodge, have shown the richness of the field. Similar studies of children have as yet scarcely passed the anecdotal stage.

In the case of animals, Lubbock found ants to be very stupid about discovering larvæ, when the latter were removed only six inches. The tracings of the paths of these ants remind one of a badly tangled mass of threads. Bees also have in varying degrees the same difficulty in finding their way. Lubbock put a bee into a bell glass 18 inches long and with a mouth $6\frac{1}{2}$ inches wide, and turned the closed end toward the window. After an hour's trial the bee failed to make its escape. Flies got out at once. Another bee tried fruitlessly for half an hour, when the open end of the jar was turned toward the window and the bee flew out at once. Bees slowly learned to find their way, after a number of experiences.[2] Hodge found that a blindfolded shepherd dog's logic of search was no more systematic than that of Lubbock's ants, while Lloyd Morgan says that dogs using both sight and smell show the same bungling researchings.

Turning now to a slightly different aspect of the question of method, we find by far the most elaborate and precise experiments upon dogs, made by Lloyd Morgan. So typical are they that it seems best to describe one of the tests somewhat in detail.[3]

The dog, Tony, is a fox terrier about 14 months old. The scene of operations is a field, along one side of which run vertical rails about six inches apart, between which the dog can readily pass. There is one place where a rail is absent, and the gap is therefore

[1] Herbert Spencer, "Principles of Biology," N. Y., 1882, I., p. 80.
[2] "Ants, Bees and Wasps," pp. 278-9.
[3] "Introduction to Comparative Psychology," London, 1894, pp. 255 ff.

wice the usual width. Along one side of the field, at right angles o that described, there is an ordinary open iron fencing.

"*First Day.*—Standing on the path adjoining the field, and separated from it by the first mentioned vertical rails, I sent the dog after a short stick into the field, and called him back through the railings. The stick caught at the ends; I whistled and the dog pushed and struggled vigorously. He retired into the field, lay down, and began gnawing the stick. I called him and he came up slowly to the railings, and stuck again. After some efforts he put his head on one side, and got the stick, a short one, through. I patted him, and showed him my satisfaction. Then I sent him after it again. He came up to the railings with more confidence, but, having the stick well by the middle, found his passage barred. After some struggles he dropped the stick and came through. I sent him back to fetch it. He put his head through and seized the stick by the middle and then pulled with all his might, dancing up and down in his endeavors; wriggling his head in the efforts, he at last got the stick through. A third time he again stuck; again dropped the stick; and again seizing it by the middle tried to pull it through. But when I sent him after it he went through himself, picked it up by the middle, and tried to push his way through, succeeding after many abortive attempts by holding his head on one side."

On the second day a short stick was first used. First time the dog brought it cigar-fashion, and when it struck a rail turned his head and brought it through. But in several succeeding trials always seized it by the middle, and struggled as on the first day; had same experience with longer stick, but soon hit upon the plan of avoiding the struggle and coming around by way of the open fence.

On the third day he showed no improvement, but more quickly shirked the problem by running around to open fence. After several weeks another trial showed no improvement.

In other experiments when the stick was so placed that it could most easily be seized in the most effective way, namely, by one end, the dog failed to take advantage of it, but seized it by the middle.

At another time the dog was given a cane heavily loaded at one end; at first he grasped it by the middle, which made the carrying of it a most awkward performance. After two hours' experience he had gradually learned to seize it at a balancing distance, nearer the heavy end. He had slowly learned to profit by his mistakes.

Hodge[1] in his study of the homing of pigeons was led to investigate the natural logic of search. Believing that those animals survive who have developed the most exhaustive methods of searching a given area for food, he sought to discover how nearly the procedure of carrier pigeons approximates to the ideal. For comparative determinations he devised the following experiment, which was chiefly tried by children and adults. A ball is so hidden in a square field that the ball can be seen when the observer is twenty feet distant. From a stake in the centre as starting-point, what is the best method of finding the ball? The mathematically best method

[1] "The Method of Homing Pigeons," *Pop. Sci. Monthly*, April, 1894.

is a path of spiral shape, the distance between the lines being 40 feet. This involves practically no researchings. Another logical method is that of a series of straight paths gridironing the field in a way. This involves the searching of some areas a second time. Simpler logical methods might be described here; but enough, perhaps, has been said to make clear the conditions of the test and the means of grading the methods. As to results, most of the adults approximated very nearly the theoretical curve. A boy of 12, however, starts for the fence, follows it for some distance, then turning in, discovers the ball by accident. His curve is somewhat logical, but naturally of lower degree than those of adults. Tests of a number of children varying in age from 3 to 12 show surprisingly little logic. The tracings of a bright six-year-old girl resembled the tracings of Lubbock's ants, revealing scarcely a trace of system, and full of researchings of areas, already searched time and again. After 75 minutes she still failed to find the ball, which adults discovered in from 4 to 12 minutes.

Experiments made by the writer upon a number of kindergarten children, by means of the "20-Question" method, also emphasizes the "hit-and-miss" nature of their logic. At first the children were tested individually. The teacher thought of some object, and as the familiar game goes, the child was to discover what it was by questioning, the questions being such as could be answered by "yes" or "no." In only one or two cases was there more than a suggestion of systematic procedure. The typical form was a series of questions about particular things: "Is it a squirrel?" and the like. The very same questions were occasionally repeated by the same child. In the few exceptional cases above noted, a general question, such as, "Has it four legs?"—"two eyes?" when answered was made little use of. For instance, the child immediately, when told it did *not* "have four legs," would ask, "Is it a bear?" Investigation of the nature of the questions shows that "recency and vividness" of concrete experiences were responsible for nearly all. There had been a squirrel in the room only a few days before, and so on. There was also a rather remarkable uniformity in the questions of the various children, indicating the narrow range of associations and memories available upon such a demand.

The test was much enjoyed by the children, and became a fixture in the program of the school, all the pupils taking part together. Gradually they began to use some methodical means and to appreciate the value of general questions. This was no doubt due in part to imitation, as the questions

gravitated quickly toward a stereotyped form, and that not the best possible. But even then, after somewhat narrowing the possibilities by the use of general questions, they tended to break away from system, and began to "plunge," by means of particular questions. The intuitive as well as the explosive nature of child thought would not down, and successes were often remarkable.

So much for animal and child method. The similarities are too obvious to need special comment. But we are brought face to face with the old question: Are these forms of reason, or can they be explained in terms of simpler processes? Lloyd Morgan, proceeding with the postulate, that no action is to be explained in terms of a higher process if it can as well be explained in terms of a lower, holds that animal procedure in general, although intelligent, is not reason. *The* method *par excellence* of dog and chick and monkey is that of sense-trial and error. The animal learns by sense experience and without the aid of the more direct path, namely, perception of relations. This perception of relations constitutes reason. True, according to Morgan's optical simile, such perception of relations is implicit in the animal's cognition, is in the indirect field of consciousness. It must come into the focus before it can be largely effective in determining procedure, and in Morgan's opinion it probably never does become focal in animals.

Another illustration may be taken from Morgan. The dog wishes to go out into the road; there is a fence between. In thrusting his head through successive spaces in the fence, as dogs are wont to do, he, by chance, strikes one space above which lies the latch of the gate. In throwing up his head the latch releases the gate. Meanwhile the dog has turned away as if to return to the house, all unconscious of his happy stroke. The noise of the swinging gate, perhaps, attracts his attention, and discovering his opportunity he rushes into the road. With remembrance of success he goes next day, and thrusts his head mechanically through many spaces until he finally strikes the right one, and again the gate opens. Now, according to the above view, if he perceived relations, if he were not the victim of his train of associations and coördinations fixed by the first day's experience, he would go straightway to the latch. But no. Three weeks of successive trials are needed to enable him to strike this space at once. Even then he continues to lift the latch with the top of his head rather than by the more economical method of using his muzzle.[1] This is one of the best illustrations of the bungling nature

[1] *Ibid.*, pp. 289 ff.

of the method of sense-trial and error. None of the links in the chain of associations lapse readily enough to facilitate readier passage to the goal. It is a process of slow and toilsome perfectioning, and indeed may never become quite perfect.

As to whether animals do not at all perceive relations, present knowledge cannot decide. Perhaps the more intelligent of them do have an awareness of simple relations and make use of them in adaptations. Romanes' Cebus[1], in his learning to unscrew and screw the handle of the hearth-brush, strongly suggests that there was some perception of relations involved. Hiram Stanley[2] has also urged that other animals are thus conscious. Certainly any attempt to draw a too sharp line of demarkation between man and brute seems to do violence to present views of mental evolution. This much seems true: some of the higher animals may have a rudimentary perception of relations. Most of the adaptations of animals are on the sense-trial and error "level."[3]

The features of this "sense-trial and error" method are briefly these: 1. Repetition of many useless movements, simply because they have been made before, and refuse to make way for a short circuit. 2. Relative slowness in profiting by errors. A number of repetitions is often necessary in order to effect any change in procedure. 3. When variations occur, they are in general relatively slight. In short, successful adjustment is the result of a large number of experiences, and is attained by a series of slight variations.

We have found the procedure of children predominantly of the "sense-trial and error" order; but that they may perceive simple relations and make simple adaptations thereby, is perhaps true. The very early appearance of this ability cannot be doubted. A case in point is the oft-quoted one of Preyer's child, who in the 17th month used a traveling bag as a foot-stool from which to reach for something.[4]

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[1] "Animal Intelligence."

[2] *Psychological Review*, III, pp. 536-41.

[3] The above view receives some neurological support from Flechsig's recent work, "*Gehirn und Seele.*" In this the author maintains that the cerebral cortex of adult man is composed not only of a number of sensori-motor centres, but also of areas of higher order, called "associational," which are assumed to be representative of the elaborative mental processes, which of course deal with "relations." In the newly born child no "*Associations-centrum*" is functional. In most animals these areas are not even present. The evidence as regards animals, while not conclusive, points in general to the distinction we have been considering.

[4] Preyer: "Development of the Intellect," N. Y., p. 12.

But most of the demands upon children are sufficiently complex to throw them back, in the absence of imitation, upon this last and deepest resource, "sense-trial and error."

As earlier suggested, in older persons also, this primitive mode has a much larger place than is often suspected. Not only excessive unfamiliarity and complexity of the new situation, but also fatigue, temporary loss of interest, a fleeting state of mental muddle may produce a relapse into the animal method. A well-known scientist reports that in dealing with complex tables and statistics, and the like, he often when tired finds himself repeating again and again some trivial or important step, long ago really disposed of. This phenomenon in the absent-minded is too familiar to need illustration.

We are now in position more fully to analyze and interpret the results of our experimental puzzle test. The problem of conscious adaptation resolves itself into two questions: 1. What is the nature of the perception of the difficulty? 2. What is the resulting method of procedure in solving the problem? First as to the nature of the conception of the problem. Introspective notes by the children not being obtainable, we must infer their notion of the problem largely from their procedure. Most began on the outside of the figure. A large number of the younger pupils drew the square first, and only after repeated trials did they press into the intricacies of the inside of the figure. From these facts we infer an incomplete comprehension of the design. How explain this inadequacy? (*a*) Recent studies of children's drawings, notably the investigations of Barnes, Lukens and Baldwin, show that children are most impressed with outlines. Any connected view of elaborate details is apt to escape them. In the above experiment the outline, *i. e.*, the square, is very familiar and simple. Some children, furthermore, thought they had traced every line, when some were really omitted. (*b*) Another factor which may have considerable significance is that the complexity of the inside of the figure may have at first repelled them. (*c*) The testimony of adults renders it extremely probable that the children, when they finally did enter the figure, were also troubled by the oddness of the middle diagonal; still very few seem to have attached to it any great significance.

As to procedure, may it not have been determined in somewhat the following manner? The outline square caught the attention and set going tendencies fixed by long habit, to draw the square, etc. This movement once made tends to be repeated. It may be a form of the circular reaction, if you please, and the child is doing just as the animal—"imitating"

his former movements, consciously or unconsciously. Repeated failures lead to a dissatisfaction with the movements, and the result is variation. But so great is the inertia of the nervous mechanism, so weak the power of inhibition, that repeated failures seem unable, in most cases, to produce radical removal of the source of error. In the light of the above facts, it may be said that the child, even so old as eight years, is rather like an animal in method of adaptation than like the human adult. The young child is a reflex and automatic organism. His narrow motor and sensory experiences enable a few tendencies and habits to rule with a potency, which yields only to time and wider experience. With age the motor habits become rich in variety, and when action is demanded tend to neutralize one another, and make for that hesitation which enables "considerations" to tilt the balance more easily in favor of the best alternative.

What, then, of the so-called "plasticity" of childhood? If the above conclusions are true, plasticity does not, as often implied, mean resource, initiative, promptness of adaptation to the new. It must mean, rather, that children are imitative beings, and hence can quickly learn new ways of doing. Every normal child may indeed be a "genius," but not of the inventive and creative sort. Just as recent researches indicate that he is less inventive in language than formerly thought, so in other phases of activity, less and less is being credited to his initiative, and more to imitation. This does not degrade the mental status of children, but rather dignifies imitation as the great means by which the mind gets experience. Inventiveness is a plant of slow growth. Protected as he is from the bewildering complexity of environment, the child only slowly gains the wide variety of experiences which favors creative activity, and which makes for the higher adaptability that is necessary for adult life.

It is a coincidence of some suggestive value that many of the characteristics of mental adaptation may be described in phrases descriptive of evolution in general. Evolution, as generally considered, is a series of small successive changes. Many facts accentuate the unequal rate of change. The extreme of this tendency is found in the theory which emphasizes "sports"[1] as the usual method of evolution. These "sports" represent a rapid summation of changes, thus supplanting *slow* unfoldment by spurtiness. Even Galton[2] admits that these "sports" *may* be conceived as accumulations of successive small increments, although he inclines to the view of large increments. Now, in almost every field of growth, spe-

[1] Cf. Galton: "Natural Inheritance," London, 1889.
[2] *Ibid.*, pp. 32 ff.

cial cases of the above are discoverable. The spurtiness of physical growth, the periods of slow unfoldment of a member or an organ, or of the whole body, followed by a rapid acceleration; in the psychophysic domain the summation of subliminal stimuli is too familiar to need further description. Moreover, in the field of conscious attention, favorable conditions, many of which are perhaps physiological, produce the same summation phenomenon. Bryan and Harter have recently shown that the improvement in learning to "receive" telegraphic messages, while rapid for a time, ceases at a point just below the required proficiency, to be followed later by a second stage of more rapid improvement.[1] This description seems generally valid for all complex mental adaptations, and suggests anew the important rôle of the subconscious physiological factors in producing occasional accelerations.

If the different methods of adaptation be arranged in a series, as follows: Sense-trial and error, Receptual, Conceptual, certain general statements seem warranted. (1) *Objectively viewed:* With approximation to the highest level, there is greater and greater acceleration of adaptation. Variations are wider, and the goal is attained more and more by leaps and bounds. Furthermore, adaptation is more *comprehensive*. This also represents acceleration. (2) *Subjectively:* There is a greater and greater activity and elaboration of central processes, conscious and unconscious. Sense-trial and error can cope successfully and promptly with only a relatively simple environment, whereas the methods of higher level are imperative for complex adjustments. The economy of time, with rise in the scale of method, measured both by gain in comprehensiveness as well as in promptness, may be found, on further investigation, to follow some such law as that of the improvement due to habituation.[2]

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[1] "For many weeks there is an improvement which the student can feel sure of, and which is proved by objective tests. Then follows a long period when the student can feel no improvement, and objective tests show little or none. At the last end of the plateau the messages on the main line are, according to the unanimous testimony of all who have experience in the matter, a senseless clatter to the student, practically as unintelligible as the same messages were months before. Suddenly, within a few days, the change comes, and the senseless clatter becomes intelligent speech."—"Studies in the Physiology and Psychology of the Telegraphic Language," Bryan and Harter. *Psych. Rev.*, Vol. IV, No. 1, p. 52.

[2] Influence of habit proper, in increasing promptness of conscious adaptations, is best seen in the operation of such generalized habits as those of the physician in diagnosing disease. Inquiry among members of this profession developed the fact that while the novice

IV.

General Conclusions. Psychic development results in conscious adjustments to outer relations, which increase in number, complexity and variety. As well known, this development is not uniform, but falls into fairly well-marked stages.

The play instincts bear teleological relations to this growth. (1) The play impulse probably arose through the fact that those animals which in play gained preliminary exercise of activities useful in maturity survived in the struggle for existence. Play also serves as an index of the nascence of certain interests, capacities and even epochs of development.

The intellectual play instincts seem ripest in the immediate prepubertal years, and may correspond to an increase in the medullation of the associative fibres of the cortex, which is the prerequisite of function of the elaborative mental faculties. Puzzles which epitomize the more complex types of relations which may produce illusion or error are most interesting at this period. Intellectual play may develop mental flexibility, versatility, and even power of will, which in turn are of manifest importance for the more exacting and intricate reactions which come with adolescence.

The line of growth in ability to cope with environment, is indicated by three roughly demarkated stadia or levels: Sense-trial and error, Receptual, Conceptual. The method employed in a given case is determined partly by the capacity of the individual, partly by the difficulty of the problem, but in general, Sense-trial and error is predominantly the level of animal and child.

In closing, one or two rather obvious connections of this study with Pedagogy and Individual Psychology may be pointed out.

If the business of Education is to help the child to acquire the most economical and adequate means of meeting the demands of increasing complexity of life, then it must provide

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slavishly follows some standard scheme of diagnosis, which is an exhaustive application of the method of exclusion, with experience, he gradually dispenses with the detailed scheme, and finally may learn to go straight to the mark as by intuition. One physician says, "When a man steps into my office, I may mentally say, without a moment's reflection, 'That man has valvular disease of the heart.' Now, if called upon to give reasons for my judgment, considerable reflection is required, and even then some salient details may not come into clear consciousness, although my diagnosis has been correct." Now, when it is remembered that physicians deal not with disease, but with sick people, and that perhaps no two cases are alike, hence offering a somewhat new problem, the gain in time is more significant.

for his induction in proper season, from the sense-trial and error level into the modes of more prompt and comprehensive adaptation. The early school years belong of course to the sense level, but the rising curve of puzzle interest marks the prepubertal age as the time to hasten transition to the higher mental methods. The young child's simple, crude, sketchy, halting, analogical modes of appreciating relations may now be safely supplanted by more rigid and logical regimen. Furthermore, if too great difficulty of task, fatigue, temporary muddle, or loss of interest, tend even in the adult, whose conceptual processes are long established, to throw the mind back upon lower levels of adaptation, the adjustment of task to pupil receives new emphasis. The hygiene and practical importance of attempting the difficult only when in the top of condition rests on this ground.

The present trend of Psychology, in many quarters, is clearly away from the simpler problems of sense and muscle, and toward the more complex and immediately interesting questions of emotion, psychogenesis, pedagogic and individual psychology, or in Wundt's phrase, "individual characterology." For the testing of some of the most important qualities of mind and character an unusually rich material is offered by the extant collection of puzzles, and a properly graded set of such tests could hardly fail to furnish much valuable data. It is very evident, from the progress already made in this field, that the simpler tests of reaction-time, memory-span and discriminative sensibility are not so well suited to bring out those individual differences that are of account in the general conduct of life as are tests more closely related to the complex activities in question.

APPENDIX.

HISTORICAL NOTES ON PUZZLES.

An inventory of puzzles shows them to cover a wide field and to be rich in number and variety. But the varieties within a single group are often reducible to a relatively small number of typical forms, many of which are of ancient origin. This paucity of type-forms and their persistence seem to be a result of a sort of a survival of the fittest. Tylor,[1] speaking of games, says: " . . . . when a game is once worked into perfect fitness for its place in the life of boys and men, it may last on with remarkable permanence, as when we see represented in the ancient Egyptian tombs the counting game, well known to us by its Italian name, "*morra;*" . . . . . Thus there is always a fair chance of finding in existence in modern times any of the popular games of the ancients." That the same statement is equally true of many puzzles, can be easily shown. Especially true is it of the Riddle.

[1] E. B. Tylor: "Geographical Distribution of Games," *Journal Anthr. Inst.*, IX.

Language Puzzles.

The Riddle:—The origin of the Riddle is not known, but it has been and is now found pretty generally even among peoples of a low degree of mental advancement. To quote Rolland:[1] "From the Vedic riddles to the riddle contests of Scandinavian gods, or of German minnesingers; from the famous question of the Sphinx to the '*Philosophies des Enigmes*' of Menestrier; . . . . ; from the riddle that caused the death of Homer to those which amused all the Wolofs,—we find great variety, wide contrasts, but nevertheless a fundamental resemblance." They play upon analogies among things perceived. Essentially, the primitive mode of invention is as follows: Some one discovers a new analogy among natural objects, formulates a question concerning it, and thus a new riddle is born. While the most primitive forms have chief reference to natural objects, the evolution of the riddle reflects the shifting of man's chief interest from external nature to man himself. Some of the most famous riddles among the Greeks have this human focus.

Just when, in the development of a people, riddle-making begins and also the period when it loses its soberer aspect and becomes a mere sport or pastime, are not easily determined. They bear obvious relation to intellectual status. Tylor thinks that the simpler forms, the "sense-riddles," belong thoroughly to the mythologic stage of thought, and are in consequence found at home among the upper savages, and range on into lower and middle civilizations. "The making of riddles," says he, "requires a fair power of ideal comparison, and knowledge must have made considerable advance before the process could become so familiar as to fall from earnest into sport. In higher states of culture, riddles begin to be looked upon as trifling. They survive only as the remnants of child's play."[2] The rôle of the riddle in education among lower races must be important. Among the Basutos "riddles are a recognized part of education and are set like exercises to a whole company of puzzled children."[3] It is training in that analogy-thinking which is indeed the true father of generalization and classification, whose high perfection is reflected in modern science. Even among people of higher culture, the ability to answer riddles was considered a proof of great sagacity. The ability to interpret some of the replies of the Greek oracles was a supreme test of wisdom.[4] Among Semitic peoples the same criterion existed. Samson's riddle, and Solomon's success in answering the questions of the Queen of Sheba, need only be mentioned. Mr. Lockhart has translated a Hebrew manuscript which claims to give the list of problems proposed by the Queen of Sheba. There are nineteen in all, and some are certainly remarkable.[5] In Märchen, and ballads of a later period, the hero's chance of winning his beloved, or of escaping threatened punishment, often turns on his power of answering riddles.

Analysis of the riddle shows it to contain some of the chief elements of literature. The anthropomorphizing and personalizing tendencies, which often characterize it, show the riddle to be closely related to the fable. And having its deepest roots in the perception

[1] Rolland: *Devinettes ou Énigmes Populaires de la France.* Edition of 1877, Paris.
[2] Tylor: "Primitive Culture," I, p. 91.
[3] *Ibid.*, I. 90 ff.
[4] "Greek Oracles," F. W. H. Myers. Hellenica. London. 1880.
[5] "The Riddles of Solomon in Rabbinic Literature," J. H. Stewart Lockhart, *Folk Lore*, I. 1890.

of analogies in nature, the riddle is brother to the metaphor, which has been so important in the development of languages and myths. A riddle may, indeed, be defined as a metaphor or group of metaphors, whose usage has not yet become common and whose explanation is not evident.

The Enigma:—While the riddle may have been important in the intellectual life of the Hebrews, its wide development and rich elaboration seem to have been reserved to the Greeks. One of its modifications, the enigma, merely a riddle in poetic form, was tried by the greatest Greek poets, and some even devoted whole poems to them, as the Syrinx, attributed to Theocritus. In the "riddle-revivals" of later periods, as we shall see, the culmination was always in an interest in enigmas.

Before proceeding to consideration of the import of riddles, it is perhaps worth while to note several other forms of language puzzles at least three of which are closely allied to the groups already discussed. *The Rebus* also bears evidence of very ancient lineage. Generally it is a form of riddle in which the problem consists in the interpretation of pictures or objects. This use of object-method obviously requires in its simpler forms no very high degree of intellectual advancement. It may well be a reverberation of man's earlier modes of symbolic expression. One of the earliest examples recorded is as follows: When the Scythians were invaded by Cyrus, they sent messengers bearing arrows, a rat and a frog; implying that unless he could hide in a hole like a rat, or in water like a frog, he would not escape their arrows. André asserts that the rebus had a considerable vogue in ancient Italy. In Via Appia, Rome, is a tomb still existing, that of a certain Publius Philo-*musus* which has on it well executed bas-reliefs of *mice*.[1] Cæsar,[2] while one of the masters of the Roman mint, placed the figure of an elephant on the reverse side of public money in defiance of law, because "Cæsar" meant "elephant" in the Punic language. Even in England to-day, complicated rebuses abound on monumental brasses, tombs and sculptures.[3] In the rebuses most familiar among us, alphabetical writing and picture writing are usually combined. *The Conundrum* is perhaps of much later origin than the rebus. It is a riddle involving a play upon words, often in the form of a pun, and thus presupposes a considerable acquaintance with language and the facile use of a somewhat elaborate vocabulary. The conundrum is said to have been a favorite source of entertainment at the later Roman feasts. It is one of the last forms of riddle to fall thus to the level of a mere pastime.

The Charade is also an elaboration of the riddle. Two chief forms are distinguishable. In one, the object referred to is described by other objects, but generally by action or gestures of persons proposing the problem. In its higher form, it resembles the enigma in that it is set in metrical form, but the riddles involved have reference to words, or parts of words, instead of objects. This form of charade, which has maintained its dignity as an amusement for adults, is probably of very recent birth, not being known in France, at least, in 1771.[4] Within two or three years several volumes of charades have been published in this country and enjoy wide popularity.

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[1] J. Lewis André: "Puns and Rebuses in History and Archæology," *Reliquary*, XXIII, p. 169.
[2] D'Israeli: "Curiosities of Literature."
[3] André: *ibid.*
[4] D'Israeli: "Curiosities of Literature."

The Anagram, although a language puzzle, partakes somewhat less of the riddle element than do the foregoing. It involves the construction of a significant word or phrase from the letters of a given name. Originally oriental, it became known to the Hebrews, who classed it among the cabalistic sciences. They believed occult properties to reside in proper names. Finally the anagram became current among the Greeks. "Plato had strange notions of the influence of anagrams, when drawn out of person's names, and later Platonists were full of the mysteries of the anagrammatic virtues of names."[1] Anagrams thus constructed from letters of the name of a given person, often even influenced his choice of vocation. This mystic significance of the anagram thus almost eclipses its puzzle element. Its later use as a puzzle, however, entitles it to some notice in this sketch.

The above seems to represent the more fundamental types, out of which most of the later forms of language puzzles have grown.

Revivals of Interest in Language Puzzles.

At certain later epochs there are traces of renewed interest in language puzzles, and especially in riddles and enigmas. Such periods seem to coincide with seasons of intellectual awakening. In the latter half of the 7th century, Adhelm, Bishop of Sherbourne, left a number of enigmas in Latin hexameter which have been repeatedly printed. Before his time there was a collection of Latin enigmas, each containing three hexameter lines. The revival thus begun propagated itself throughout the remainder of the Anglo-Saxon period. There were eighty riddles and enigmas in English before the Norman conquest. In Protestant countries the Reformation put a stop, for a time, to the riddle-making.

In the 17th century, a great riddle era in France, there was considerable publication. At this time, Menestrier also wrote his "Philosophy of Enigmas." The taste spread to England. Swift, Cowper, Fox and others wrote a number. One of the famous prose riddles of this period was that by Fox, on a "Watch." "I went to the Crimea; I stopped there, and I never went there, and I came back again." More than a score of Swift's enigmas are printed.[2] Some are of considerable length, notably these: "On a Pen," "On a Corkscrew," "The Gulf of all Human Possessions," "An Echo," "On a Shadow in a Glass." One of the best of the briefer ones—that "On Time"—is as follows:

> "Ever eating, never cloying,
> All-devouring, all-destroying,
> Never finding full repast,
> Till I eat the world at last."

Again, in the 18th century, notably in France, the interest in riddles and enigmas became marked, Voltaire and Rousseau writing, among others. "*Mercure de France*" became a repository of riddles and enigmas, the solution of which was sufficient to make a reputation in society.[3] To such exuberance of mental energy, and especially the delight in playing with language, is to be ascribed the invention of other intricate word puzzles, as the acrostic, word-square, charade, and the like. Boccaccio wrote a giant acrostic of fifty cantos.[4] During the Elizabethan age, words and verses were tortured into the most fantastic forms. Acrostics and chronograms were much in vogue.

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[1] D'Israeli: "Curiosities of Literature."
[2] Works of Jonathan Swift; edited by Sir Walter Scott, Vol. XV, London, 1883.
[3] Art. "Riddles." Chambers' Encyc.
[4] D'Israeli: *ibid.*

Summary. Language puzzles, and especially riddles, arising from the same soil that produces literature and mythology, seriously engage the mind of peoples of relatively low degree of intellectual advancement. They depend upon perception of analogies, which is the predominant characteristic of the thinking of early peoples and of children.[1] The riddle probably plays a serious rôle in education among lower races. But later, when perception of analogies becomes sufficiently easy, these puzzles lose their soberer aspects and serve merely as amusements. The later development of language puzzles shows the influence of growing culture. The riddle makes way for the enigma, with its regard for literary form; for the conundrum, which savors of the three R's; and finally for the acrostic, word-square and charade, which take one farther and farther from the natural objects that were the original theme of riddles.

Mathematical Puzzles.

The germs of mathematical puzzles must have appeared early in the development of mathematics, but the first works devoted to them date from the 16th century. The list of works since that time, containing either a general collection of these, or special consideration of single problems, numbers upwards of 120. De Bouvelle's "*Propositions arithmeticæ ad acuendos juvenes*," printed 1543, (credited at different times to Bede or Alcuin) is said by Lucas to be the first known beginnings of mathematical recreations. Perhaps the best known and most popular works of this class, however, are Bachet's[2] "*Problemes plaisans et délectables*," Paris, 1612, which has gone through several editions, the last two in 1874 and 1879 respectively; and Ozanam's "*Récréations mathématiques*," Paris, 1694, of which also a good many editions have been sold, and which has been much revised and popularized by Montucla. A translation into English of Montucla's edition was published by Charles Hutton early in this century. A later edition bears the date of 1840. These works of Bachet and Ozanam are the chief sources of the modern mathematical puzzles which appear in popular works on the subject, and in juvenile periodicals. Two recent works deserve mention: Ball's "Mathematical Recreations and Problems"[3] and Lucas' *Récréations mathématiques*,"[4] in five volumes. A long line of distinguished mathematicians, including Cardan, Tartaglia, Fermat, Leibnitz, Euler, Listing, Plateau, Thompson, Sylvester, Story and Ch. Henry, have interested themselves in problems in this field. Several histories of the attempts at the "Quadrature of the Circle" and of the forms of "Magic Squares," bear abundant testimony to the fascination which these problems possess for the mathematical mind. "Magic Squares," moreover, were invested with a mystic signification which long endured. Albrecht Dürer, in his famous "Melancholy," engraved in 1514, has represented a magic square, showing that even in his day, it retained at least a symbolic meaning.

The 14-15-16 puzzle, so much in vogue within comparatively recent years in this country, is said to have been invented by an Englishman 200 years ago. It has been mathematically treated by Story and others. Puzzles of the labyrinthine and maze types, so

[1] Cf. Jastrow: "Natural History of Analogy." Proceedings Am. Ass'n Advancement of Science, 1891.
[2] Cf. Bibliography in Lucas' work.
[3] London, 1892.
[4] Paris, 1882.

familiar in puzzle columns, have a history. These forms have always fascinated men. The famous labyrinths of ancient times—the Egyptian, Cretan and Samian—were reckoned among the wonders of the world, and were held to be unthreadable. The Egyptian labyrinth contained more than 3000 chambers. In the middle ages the custom of constructing in the walls and pavements of churches labyrinthine designs, is well known. The same pattern also came to be a feature of landscape gardening, and the labyrinth of Hampton court is one of the most famous in England. Mathematical interest in the labyrinth in all its varieties, and also its vogue as a puzzle, probably dates from the middle ages. Tremaux and others have published rules for threading such figures.

These instances illustrate sufficiently, perhaps, how many puzzles have from early times strongly appealed to the human mind. It is hardly a chance coincidence that so much of myth and mystic meaning has clustered about them, in view of their peculiarly baffling quality.[1] On the other hand we have seen how some problems (as the 14-15-16 puzzle), although originally invented to amuse, may by competent mathematical treatment be lifted to the plane of a real and perhaps important problem, and thus contribute in a direct way something to the enrichment of science.

Mechanical Puzzles.

The history of this group is even more meagre than that of the preceding. Aside from those mechanical puzzles whose mathematical tractment has been exploited, very little account is obtainable. It may be expected, however, that anthropological investigation will show the same wide distribution and similarity of origin that have been found in the case of games. The University of Pennsylvania's exhibit at the Columbian Exposition contained 129 mechanical puzzles, most of which are of oriental origin.[2] The larger number are not, as commonly believed, invented in China and Japan, but rather in India. Sporadic invention of puzzles has indeed occurred in modern times in Europe, while America has made something of a name for the ingenuity of its inventors in this line. But few are entirely original, and the usual *modus operandi* of the puzzle maker is to graft onto an old type enough slight modifications to give the appearance of novelty, and commend the whole to the public by a fetching name. The distinct types of puzzles are few in number.[3]

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[1] To illustrate how myth and legend tend to cluster about puzzles, the following concerning the familiar mechanical problem, the Tower of Hanöi, is in point. M. De Parville gives an account of the origin of the toy. "In the great temple of Benares, beneath the dome which marks the centre of the world, rests a brass plate in which are fixed three diamond needles, each a cubit high and as thick as the body of a bee. On one of these needles, at the creation, God placed sixty-four discs of pure gold, the largest disc resting on the brass plate, and the others getting smaller and smaller up to the top one. This is the tower of Bramah. Day and night, unceasingly, the priests transfer the discs from one diamond needle to another, according to the fixed and immutable laws of Bramah, which require that the priest must not move more than one disc at a time and that he must place this disc on a needle so that there is not a smaller disc below it. When the sixty-four discs shall have been thus transferred from the needle on which, at the creation, God placed them, to one of the other needles, tower, temple and Brahmins alike will crumble into dust, and with a thunder-clap the world will vanish . . . ."—Ball, "Mathematical Recreations," p. 78.

[2] Stewart Culin, *A. J. Folk Lore*, VI, 22.

[3] Perhaps nowhere are some of the characteristics of the problem-solving instinct more clearly exemplified, and the essential unity of the human mind, in spite of the differences of time, nationality or race, more clearly indicated, than in the history of attempts to solve a peculiar group of problems whose insolvability has called down upon them the opprobrious epithet of "Follies of Science." Prominent among these

Logical and Philosophical Puzzles.

Although Zeno of Elea, the "father of dialectic," is indirectly perhaps the first important source of puzzle material of this sort, its chief development falls in two later epochs, one ancient, the other mediæval, namely, among the sophists and the schoolmen. Both agree in exalting the formal aspect of thought, and acquired marvelous skill in the handling of phrase and proposition, which, as it degenerated, often sunk matter in method and made discussion a rhetorical swordplay.

Two of the minor sophists, Euthydemus and Dionysodorus, are of this sort. Plato draws a memorable picture of them in his "Euthydemus."[1] Claiming to be masters of the art of eristic, or fighting with words, they stand ready to teach for a consideration. A few of the questions which were put to the youth Cleinias give a fair idea of their expert word-juggling. "Cleinias," says Euthydemus, "who learn, the wise or the unwise?" "The wise," is the reply. . . . "And yet when you learned you did not know and were not wise." Again: "And do they learn what they know or what they do not know?" "The latter." "And dictation is a dictation of letters?" "Yes." "And you know letters?" "Yes." "Then you learn what you know." "But," says Dionysodorus, "is not learning acquiring knowledge?" "Yes." "And you acquire that which you have not got already?" "Yes." "Then you learn that which you do not know." And so it goes. If we wonder how any people should have taken such word-juggling seriously, we should remember the status of the Greek mind. Every philosophic idea was in a state of flux; contradictions were rife; logic was not yet written; there was no analysis of grammar. Language was first beginning to perplex human thought. The contribution of the sophists to the intellectual activity of the Greeks was genuinely important. It is now admitted that they led to the systematic study of grammer, rhetoric, philology, and were the necessary propædeutic for the logic of Aristotle.

The most interesting and ingenious arguments of the sophists survive in the logical treatises, but any attempt to classify the whole range of their puzzles would be futile. Many depend upon ambiguity of meaning of terms. Others are of more intricate logical form.

The puzzle interest in Greece did not die with the sophists. According to Diogenes Laertius, Chrysippus, the stoic, wrote six different treatises upon Eubulides' famous puzzle, "The Liar," and Philetas of Cos studied himself to death in attempting to solve it.[2]

Three illustrations will sufficiently exemplify the sources of the puzzles of this early period. Zeno's arguments against the reality of motion, of time, space, the manifold, and the veracity of sense perception, can be characterized, as to form, by a single one against motion. Motion cannot begin, because a body in motion cannot arrive at another place until it has passed through an unlimited number of intermediate places.[3] Of a later period the following are representative: — The Protagoras-Eualthus argument. Eual-

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are Duplication of the Cube, Trisection of the Angle, Quadrature of the Circle, Astrology, Alchemy, and Perpetual Motion. Most are of great antiquity and have a voluminous history, but Perpetual Motion has received little attention historically. No prolonged or systematic search for the origins of this problem seems to have been made. From data already gathered, there are evidences of a close relation of this mechanical idea to certain biological and philosophical concepts; and in yet other respects, it offers a field for profitable psychological study.

[1] Jowett's edition of Plato.

[2] Bowen's "Logic," p. 289.

[3] Ueberweg: "History of Philosophy," I, p. 57.

thus received lessons in rhetoric from Protagoras, it being agreed that a certain fee should be paid if the pupil were successful in the first cause he pleaded. Eualthus neglected to take any case and Protagoras sued for his fee. Eualthus defended himself in court and it was consequently his first case. Protagoras argued, "If I be successful in this case, O Eualthus, you will be compelled to pay by virtue of the sentence of these righteous judges; and should I be even unsuccessful you will then have to pay me in fulfillment of your original contract." Eualthus replied, "If I be successful, O master, I shall be free by the sentence of these righteous judges; and even if I be unsuccessful, I shall be free by virtue of the contract." The "Syllogismus Crocodilus," Eubulides' "Liar," "All rules have their exceptions," are a few of the numerous puzzles of this group. Finally, there is a group which plays upon the manner of asking questions, so bringing it about that either an affirmative or negative involves one in apparent admissions of a damaging nature. "Have you left off beating your father?" is obviously of this sort.

Among the schoolmen is seen the apotheosis of formal logic. The mission of scholasticism was to furnish a rational basis for the Christian faith. "They burden themselves with the weight of a logical instrument which Aristotle created for theory and not for practice, and which ought to have remained in a cabinet of philosophical curiosities without ever being carried into the field of action."[1]

Disputation was the great means of education. Dexterity in framing and solving sophisms was reckoned a scholarly accomplishment and one of the special fruits of a university training. In spite, however, of the indictment of Milman[2] and others, that these activities never had nor cared to have any bearings on the life and practical opinions of mankind; in spite of their failure to add directly a single new idea to science,—nevertheless the mind of semi-barbarous Europe was thereby trained for the vast work of the modern world.

This being as it may, there can be no doubt about the arid subtleties of its decadence. Even St. Thomas Aquinas—the "greatest giant between Aristotle and Newton" — is carried away by his devotion to logic. In his *Summa Theologiæ* — where everything is thrown into Aristotelian form—are found discussions and logical demonstrations of such propositions as these: "Angels are composed of action and potentiality." "Every angel differs from every other in species." "The bodies assumed by angels are of thick air." "Many angels cannot be in the same place." "The velocity of an angel is not according to the quantity of his strength, but according to his will." "The motion of the illumination of an angel is three-fold, or circular, straight and oblique." Others discussed whether the angel Gabriel appeared to the Virgin Mary in the shape of serpent, dove, man or woman. Young or old? In what dress? Garment white or of two colors? Linen clean or foul, etc.? What was the color of the Virgin Mary's hair? Was she acquainted with the mechanic or liberal arts?

Through more than a century thousands debated the problem, "When a hog is carried to the market with a rope about his neck, which is held at the other end by a man, whether the hog is carried to the market by the *rope* or by the *man*?" The "free-will" discussion gave rise to famous problems. One of these was invented

[1] Taine: "History of English Literature," I, pp. 214-15.
[2] Milman: "Latin Christianity," VIII, p. 267.

by J. Buridan[1] (died about 1358), and is widely known as "Buridan's Ass." "An ass is equally pressed by hunger and by thirst; a bundle of hay is on one side, a pail of water on the other. Will he die for want of both, or will he make a choice?"

Remembering that a puzzle is a problem whose solution is an end in itself—a problem attempted mainly for the pleasure derived from the activity — the question naturally rises, are the above fairly called puzzles? Were they not dealt with because they had application to real questions and principles of philosophy and morality? It cannot be denied that most were originally of that sort. Aquinas certainly cannot be charged with admitting to discussion any matter which does not bear directly upon the serious task in hand. Even the apparent trivialities were important in his logical arches. It is when torn from their setting and employed as theses at a later period, when logic had fallen to the plane of a mere gymnastic, that they degenerate into mere puzzles.

Logical puzzles of this sort find few devotees to-day, though some who have reached that stadium in the study of logic and metaphysics which brings a certain mastery, find a temporary pleasure in them. A more distinctly modern species of logical puzzle grows out of the study of logical theory. The following is quoted from the "Life of De Morgan": "(1) For every z there is an x which is not y. (2) Some y's are z's. Some x's are not z's."[2] Prof. Jastrow[3] has also recently published one of this sort: "Granted that A is B, to prove that B is A. B (like everything else) is either A or not A. If B is not A, then by our first premise we have the syllogism: A is B: B is not A: ∴ A is not A; which is absurd, therefore B is A."

Whether any of the larger present problems of philosophy, epistemology, ontology and metaphysics, often take on the puzzle *quale*, may well be doubted, but that they occasionally do so in some minds seems likely. That such problems are attempted at first simply for the pleasure of the activity, is hardly possible. As Kant says: "It is in vain to assume a kind of artificial indifferentism in respect to inquiries, the subject of which cannot be indifferent to human nature."[4]

But what shall be said of the fascination which the antinomies exercise upon some minds? What of the devotees of ontology, who, refusing to accept the verdict of critical philosophies, press on by old and fruitless methods to the ultimate reality, to the absolute? It must be conceded that as the original impulse fails and as the solution of ultimate questions seems more and more remote, there appears to come in some minds a tendency to manipulate the cherished formulæ, just as the mathematicians often do theirs, to see what will "come of them," or to keep the dialectic muscles in the top of condition.

Dilemmas of Etiquette, Law, Strategy, Ethics, Etc.

To the majority of mankind the supremely interesting problems are those dealing with practical life. But for the very reason that they have a practical focus, they are not strictly classifiable as puzzles. Hence little more can be done than to suggest in the briefest and most fragmentary way a few lines along which are discoverable cases, which seem to possess a modicum, at least, of puzzle quality.

[1] De Morgan: "Budget of Paradoxes," p. 28.
[2] *Knowledge*, III. p. 7.
[3] *Science*, N. S., V, p. 105.
[4] "Critique of Pure Reason," Introduction, Trans. by Müller, p. XXI.

Mason[1] quotes the observation of a whaling captain, that the Eskimo often go out, in sport, to difficult places, and having imagined themselves in certain straits, compare notes as to what each one would do. Fifteen years ago the puzzle mania in England, after a long vogue of acrostics and the like, finally went over to personal dilemma puzzles, not unlike those of the Eskimo. They ran like this: "A certain man does so and so, and in consequence finds himself in such and such a delicate moral situation, what shall he do?" The popularity of certain stories which pose a dilemma, as the "Lady or the Tiger?"; and so-called mystery stories, a device of modern newspaperdom, wherein the culmination of the plot is left to the ingenuity of the reader, are further illustrations in point. The correspondence columns of many periodicals to-day also show the almost morbid degree of interest, on the part of some, in questions of etiquette. How many of these problems belong strictly to the puzzle field cannot be determined.

In the field of strategy the puzzles are perhaps few, but of exceeding interest to some minds. One of the writer's friends occasionally whiles away a tedious quarter of an hour by planning an impenetrable fort, and then attempts to make his way in.

From the very nature of Casuistry as an application of reason to particular cases where conflicting or apparently conflicting duties are involved, it became the source no less than scholasticism of formal distinctions, of logical subtleties, which are the rich soil for the possible growth of puzzle material. But, as in the case of philosophy, it is only in its degenerate states, and not always then that its problems are fairly to be called puzzles, having in general entirely practical ends. The later works on casuistry (up to 1700) contain such problems as these, "Does a man who steals four shillings commit a mortal sin, or only a venial one?" "Does a man who blasphemes twenty saints at once, commit twenty sins, or only one?" The culmination of this rank development is perhaps in the doctrine of Probabilism, which rested on the theory that the moral law does not bind in cases where it is doubtful, and that it may be considered doubtful if theologians of name have denied that it binds in particular cases. This was an obvious opportunity for endless discussion, and the moral laxity which the principle permitted was so effectively branded by Pascal in his "Provincial Letters" as to lead to the condemnation of the doctrine and the temporary overthrow of the Jesuits in France.

Civil law also furnishes nearly the same favorable conditions for subtle distinctions, and most of the decisions of courts are often difficult applications of principles to particular cases, and hence a form of casuistry in the original meaning of the term. The development of Roman law, with its necessarily great emphasis on definition and fine shades of meaning, must have been also especially responsible for that love of logic-chopping which is supposed to survive to-day in the law courts.

Enough has been said, perhaps, to make it patent that questions of law, strategy and ethics may degenerate into mere puzzles, and when they do so, belong to the Logical and Philosophical groups.

It remains to speak of the abnormal and genuinely morbid aspects of the mental state of puzzle. Nowhere is an abnormal turn so likely as in the field of the ethical and practical. The type of man or woman constantly brooding over fine distinctions of motives and conduct, full of forebodings as to the outcome of simple acts, which, at most, are of little real consequence, and which

[1] "Origin of Inventions."

should have been relegated to automatism, is too sad and too familiar in literature and in life to need illustration. The perspective of life is lost, and the painful unrest tends to color in sombre hue the whole stream of thought.

In distinctly morbid cases also (*Grübelsucht*, or Insanity of Doubt), the matters over which the patients ponder are those discussed in the last two sections. "One patient doubts everything, even his own existence, and is totally unable to arrive at any definite conclusion on any subject. Another cannot discuss a subject without indulging in the tiresome process of hair-splitting, and in so doing exhausting all the subtleties of scholasticism concerning matters more or less familiar or hackneyed." On the basis of the ideas which prevail in the minds of these unfortunates the following classification has been made: The *Metaphysicians*, who ponder abstruse questions. Who created God? What is the origin of language? What is immortality? and the like. The *Realists*, who think of more trivial questions. A Russian prince, for instance, wonders "why men are not as tall as houses." Another wonders why there is only one moon, and not two. The *Scrupulous*, who constantly struggle for precision in statement. They weigh fine distinctions in order to be truthful. The *Timid*, who speculate about personal or bodily accidents and their consequences. The *Reckoners*, full of anxiety to know the numbers of things, who count buttons, windows and every conceivable object.[1] Like the normal philosophical and ethical examples, these questionings, in so far as they are undertaken for supposed ends of conduct, would be excluded by a rigid definition of puzzles, but in so far as they are undertaken merely from an undefined impulse to reach a solution, they would come fairly within.

[1] Ball: "Insanity of Doubt," Tuke's *Dictionary Psych. Med.*

THE VALIDITY OF THE PSYCHOPHYSICAL LAW FOR THE ESTIMATION OF SURFACE MAGNITUDES.

By J. McCrea, B. A., and H. J. Pritchard, B. A. (Toronto).

The Psychophysical Law has been tested as to its application, for linear magnitudes, but hitherto little has been done to find out to what extent it may be applied to the estimation of surfaces. This paper gives the results of a series of experiments which have been made for the purpose of testing the validity of the law when applied to the estimation of surface magnitudes. The experiments have been performed in the Psychological Laboratory of the University of Toronto, during the session of 1896-97.

The apparatus used, constructed under the direction of Dr. Kirschmann, is essentially that used by Mr. J. O. Quantz, B. A., when experimenting on a problem akin to the present investigation during the session of 1893-94. For a description of this apparatus the readers are referred to Vol. VII, No. 1, of this Journal, to the article on "The Influence of the Color of Surfaces on our Estimation of their Magnitudes." Except for one or two minor changes the arrangement was the same as in Mr. Quantz' experiments. In the present investigation the colored surfaces were replaced by white, which were obtained by simply leaving the surface in the diaphragm open and covering the open end of the apparatus with white tissue papers in addition to the ground glass of the window.

The objects used in our case were taken from a set of 180 brass diaphragms, varying in diameter from 6 to 50 mm., which were constructed according to the device of Dr. Kirschmann, by means of a conical drill which bored through the above stated number of thin sheets of brass, which were tightly clasped together. By this means the necessity of any further finishing, such as grinding or filing, which would have spoiled the continuity of the transitions, was prevented. In order to secure a constant distance of the periphery of the circular diaphragm from one of the margins of the brass plate, the plates were set in such a way that one surface of the parallelopipedon, which they formed, was parallel to the cutting edge of the conical drill.

From this set of diaphragms we selected for each experiment two discs of different size. This was done in order to eliminate the error which might arise from knowing the posi-

tion in which the discs had actually the same visual angle. The difference between their diameters which was most convenient for us to use was about one millimeter. The measurement of the diameters of these was made with a micrometer caliper, by which it was possible to measure distances to $\frac{1}{50}$ of a millimeter. Each observer measured each disc 10 times, and the average of the 20 trials was taken.

As the discs were made with a conical drill, the diameter of the aperture on one side was somewhat smaller than that on the other. The side on which the diameter was the smaller was blackened, so as to prevent the reflection of light as much as possible. Then the diaphragms were placed on the screens, with the blackened side toward the eye of the observer.

With the same micrometer caliper also was ascertained the distance between the two discs when in the same plane. This distance remained approximately constant for all the series. The distance between the fixed or normal disc and the eye of the observer was 1230 mm., and this remained constant throughout.

In the observations, the mode of operation was the same as that described by Mr. Quantz in his paper. Each observer made 100 observations with each eye in every series of experiments, but they were not made consecutively. Only 20 or 30 observations with each eye were made by an observer in succession, and in these he made 10 observations with each eye alternately. This change was made so that the eye would not become wearied by too prolonged use. These 100 observations with each eye are called a series.

After each observer had made 100 observations with each eye, the discs were interchanged and the same mode of procedure was followed. These observations are called the second series and these two series of observation are called a set. As will be seen from the accompanying table, six such sets of experiments have been performed.

Although the intention was to apply the Method of Average Errors, the ordinary course of procedure in the calculation was not followed; for, instead of the pure average error, on the advice of the director of the laboratory, we computed the Mean Variation. The Mean Variation has the advantage of being entirely independent of the normal magnitude, thus giving only the average deviation from an ideal normal magnitude, represented by the average value of the observations. If, to the determination of the normal magnitude, an error of $\frac{1}{10000}$ would adhere, this error would be implied in each observation, and if the observations were all in one direction, either positive or negative, the error would enter the last result multiplied by the number of cases. Further, by using the Mean Variation, one escapes the ambiguity which is

involved in the ordinary use of the Average Error Method, viz.: in the Method of Pure Average Errors, we can change, without detriment to the pure average error, one observation of a series (*e. g.*, from the positive to the negative) without changing any other. This is not possible if we take the mean variation.

In employing this method, it was applied to the distances of the variable stimulus from the normal stimulus. First, the average distance between them was found and the mean variation in the 100 observations computed. Then the average distance was added to, or subtracted from, the 1230 mm., according to the direction from the normal stimulus in which the average distance was found to be. The result thus obtained was called r. The mean variation was added to r, and the result called r^a; it was likewise subtracted from r, and the result called r_1. Then by means of a trigonometrical solution, the value of the visual angles, subtended by the diameter of the disc at each of these three distances, was ascertained. The computation is exactly the same as in the case of Mr. Quantz, to whose geometrical representations the reader is referred. Then, having found the difference between the angle subtended by the disc at the distance r and each of the other two angles, the average of these two differences was taken, and this average is regarded in our tables as the mean variation. The relation between this variation and the visual angle of the diameter of the disc at the distance r was expressed in percentage; and the averages were taken of the per cent. for both of the eyes in both series of the set.

From this percentage of the mean variation of the diameter, the percentage of the mean variation of the surface magnitude may be deduced. By means of the following algebraic process, it was ascertained that the relation between these two percentages remained constant.

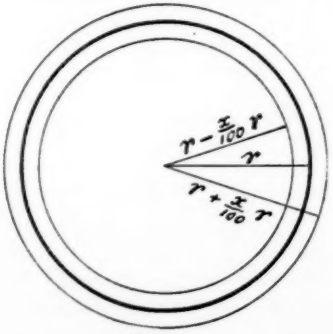

FIG. 1.

r = radius of circle, which represents the estimated value.
x = the % of r, which represents the mean variation.
$(r+\frac{x}{100}\,r)$ and $r-(\frac{x}{100}\cdot r)$=the radii of the circles between which the variations of the circle have play.

1. Area of Estimated Value $=\pi r^2$
2. Area of inc. circle $=\pi(r+\frac{x}{100}.r)^2=\pi r^2(1+\frac{2x}{100}+\frac{x^2}{10000})$
3. Area of dec. circle $=\pi(r-\frac{x}{100}.r)^2=\pi r^2(1-\frac{2x}{100}+\frac{x^2}{10000})$

Diff. of 2 from estim'd value $=\pi r^2(1+\frac{2x}{100}+\frac{x^2}{10000})-\pi r^2$
$=\pi r^2(\frac{2x}{100}+\frac{x^2}{10000})$

Diff. of 3 from estim'd value $=\pi r^2-\pi r^2(1-\frac{2x}{100}+\frac{x^2}{10000})$
$=\pi r^2(\frac{2x}{100}-\frac{x^2}{10000})$

Av. of these var't'ns $=\frac{\pi r^2(\frac{2x}{100}+\frac{x^2}{10000})+\pi r^2(\frac{2x}{100}-\frac{x^2}{10000})}{2}$
$=\pi r^2(\frac{2x}{100})$

$\therefore$ the per cent. of the variation to the estimated value (πr^2) is $2x\%$, *i. e.*, the per cent. of the mean variation of the surface magnitude is always double the per cent. of the mean variation of the diameter of estimated value of the circle.

In the tables which we give with this paper, we refer only to the diameter, but if we wish to ascertain the variation of the area, we merely require to multiply the former percentage by 2, as we have seen from the foregoing calculation.

In the experiments which have been performed, six different pairs of discs have been taken. The sizes of these discs are shown in the diagram (Fig. 2) and the results of the experiments are given in the table below.

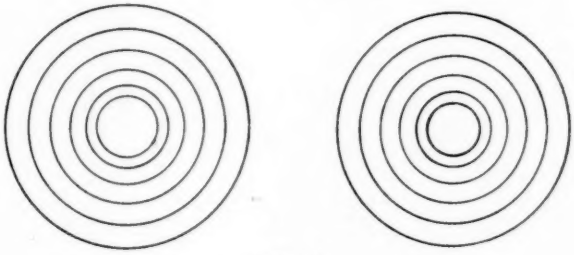

FIG. 2.

The first four columns of the table explain themselves. In the 5th is given the measurement of the diameter of the normal stimulus, *i. e.*, of the disc which remains fixed throughout the series. In the 6th column is given the visual angle

TABLE I.

Set.	Observer.	Series.	Eye.	Measurement of Normal Stimulus.	Visual Angle of Normal Stimulus.	Estimated Value.	Mean Variation.	% of Estimated Value.	Average %.	Constant Error.	Average of Constant Errors
I	McC.	1	Left	35.902 mm.	1° 40′ 14.01″	1° 36′ 34.73″	1′ 18.2″	1.34%		−3′ 39.28″	
			Right			1 36 13.62	1 12.92	1.26	1.3125%	−4 0.39	−1′ 29.1275″
		2	Left	37.306 mm.	1 44 9.26	1 45 14.95	1 28.31	1.24		+1 5.69	
			Right			1 44 46 73	1 28.76	1.41		+0 37.47	
	P.	1	Left	35.902 mm.	1 40 14.01	1 39 35.63	1 1.265	1.02		−0 38.38	
			Right			1 39 23.05	1 2.715	1.05	1.0975	−0 50.96	+1 0.1825
		2	Left	37.306 mm.	1 44 9.26	1 47 37.63	1 22.375	1.27		+3 28.37	
			Right			1 46 10.96	1 7.18	1.05		+2 1.70	
II	McC.	1	Left	30.155 mm.	1° 24′ 12.3 ″	1° 27′ 30.66″	1′ 11.67″	1.36%		+3′ 18.36″	
			Right			1 26 50.98	1 7.18	1.29	1.2975%	+2 38.68	+0′ 33.3525″
		2	Left	28.958 mm.	1 20 51.67	1 19 10.14	0 56.98	1.19		−1 41.53	
			Right			1 18 49.57	1 4.18	1.35		−2 2.10	
	P.	1	Left	30.155 mm.	1 24 12.3	1 26 56.53	0 56.09	1.07		+2 44.23	
			Right			1 26 7.88	0 51.325	0.99	0.975	+1 55.58	+1 5.5275
		2	Left	28.958 mm.	1 20 51.67	1 20 35.6	0 43.24	0.89		−0 16.07	
			Right			1 20 53.3	0 46.095	0.95		+0 1.63	

TABLE II.

Set.	Observer.	Series.	Eye.	Measurement of Normal Stimulus.	Visual Angle of Normal Stimulus.	Estimated Value.	Mean Variation.	% of Estimated Value.	Average %.	Constant Error.	Average of Constant Errors
III	McC.	1	Left	22.73 mm.	1° 3′ 30.66″	1° 1′ 28.2 ″	0′ 43.4 ″	1.17%		−2′ 2.46″	
			Right			1 1 25.72	0 50.88	1.38	1.3925%	−2 4.94	−0′ 3.6075″
		2	Left	23.923 mm.	1 6 48.58	1 9 6.08	1 6.40	1.60		+2 17.50	
			Right			1 8 24.05	0 58.53	1.42		+1 35.47	
	P.	1	Left	22.73 mm.	1 3 30.66	1 3 2.37	0 43.55	1.15		−0 28.29	
			Right			1 2 53.81	0 43.96	1.16	1.1775	−0 36.85	+0 45.4075
		2	Left	23.923 mm.	1 6 48.58	1 9 24.66	0 54.06	1.29		+2 36.08	
			Right			1 8 19.27	0 45.78	1.11		+1 30.69	
IV	McC.	1	Left	16.245 mm.	0° 45′ 22.68″	0° 45′ 9.71″	0′ 47.89″	1.76%		−0′ 12.97″	
			Right			0 45 20.87	0 41.77	1.53	2.135%	−0 1.81	+2′ 2.2875″
		2	Left	17.521 mm.	0 48 56.39	0 53 20.74	1 21.33	2.54		+4 24.35	
			Right			0 52 55.97	1 26.10	2.71		+3 59.58	
	P.	1	Left	16.245 mm.	0 45 22.68	0 45 47.6	0 47.525	1.72		+0 24.92	
			Right			0 45 34.28	0 45.31	1.66	1.7325	+0 11.60	+0 51.04
		2	Left	17.521 mm.	0 48 56.39	0 50 33.89	0 50.515	1.66		+1 37.50	
			Right			0 50 6.53	0 56.93	1.89		+1 10.14	

TABLE III.

Set.	Observer.	Series.	Eye.	Measurement of Normal Stimulus.	Visual Angle of Normal Stimulus.	Estimated Value.	Mean Variation.	% of Estimated Value.	Average %.	Constant Error.	Average of Constant Errors
V	McC.	1	Left	12.6 mm.	35′ 11.92″	36′ 2.29″	55.57″	2.57%		+0′ 50.37″	
			Right			36 16.07	43.35	1.99	2.105%	+1 4.15	+23.615″
		2	Left	11.601 mm.	32 24.53	32 11.99	37.78	1.96		−0 12.54	
			Right			32 17.01	36.90	1.90		−0 7.52	
	P.	1	Left	12.6 mm.	35 11.92	36 13.83	40.845	1.88		+1 1.91	
			Right			35 43.92	40.205	1.88	1.5775	+0 32.00	
		2	Left	11.601 mm.	32 24.53	32 40.38	27.22	1.38		+0 15.85	+25.2375
			Right			32 17.42	22.545	1.17		−0 9.81	
VI	McC.	1	Left	8.289 mm.	23′ 9.44″	22′ 26.27″	17.02″	1.26%		−0′ 43.17″	
			Right			22 31.08	17.37	1.29	2.1125%	−0 38.36	+37.7975″
		2	Left	9.291 mm.	25 57.38	27 42.59	42.81	2.58		+1 45.21	
			Right			28 4.89	55.94	3.32		+2 7.51	
	P.	1	Left	8.289 mm.	23 9.44	22 47.84	18.725	1.37		−0 21.60	
			Right			22 52.48	18.665	1.36	1.985	−0 16.96	
		2	Left	9.291 mm.	25 57.38	27 55.69	43.12	2.57		+1 58.31	+46.0175
			Right			27 41.7	43.89	2.64		+1 44.32	

SUMMARY TABLE.

Set.	Series.	Measurement of Normal Discs.	Average.	Average %.	
				J. McCrea.	H.J.Pritch'd.
I	1	35.902 mm.			
	2	37.306 mm.	36.604 mm.	1.3125	1.0975
II	1	30.155 mm.			
	2	28.958 mm.	29.5565 mm.	1.2975	.975
III	1	22.73 mm.			
	2	23.923 mm.	23.3265 mm.	1.3925	1.1775
IV	1	16.245 mm.			
	2	17.521 mm.	16.883 mm.	2.135	1.7325
V	1	12.6 mm.			
	2	11.601 mm.	12.1005 mm.	2.105	1.5775
VI	1	8.289 mm.			
	2	9.291 mm.	8.79 mm.	2.1125	1.985

The order in which these sets were observed is as follows: IV, V, III, II, VI, I.

subtended by that diameter. In the 7th appears the value of the visual angle, when it was judged to be equal to the normal stimulus. The 8th and 9th columns will be understood from what has already been said in this paper. By Constant Error is meant the difference between the estimated value and the normal magnitude.

The sets in the table are arranged according to the relative sizes of the discs used, beginning at the largest, and not according to the order in which the observations were made.

In a comparison of the results which are given in the Summary Table, it will be noticed that the percentages, although not nearly equal, still show a certain approximation to constancy. Whilst the normal magnitudes vary between extremes, which are to each other about 1:4, the per cent. of the average error varies for Observer M. only between the limits of $1:1\frac{2}{3}$ and for Observer P., 1:2. Thus the Law of Weber does not seem to hold exactly for surface magnitudes; but the results show a decided approach towards it. It will be observed, however, that with a certain degree of

regularity the percentage increases as the magnitude of the object decreases. But we see that the percentage of the mean variation in Set IV is greater than this regularity of increase would demand. This may be accounted for by the fact that this was the first set of observations made, and the accuracy of judging has probably increased with a year's practice. In Set II, where the average stimulus had a visual angle of about 1° 20′, the mean variation was the smallest; and as the visual angle decreased in size from that magnitude, the mean variation increased, as is shown in the results obtained from Sets III, IV, V, and VI. It was noticed, during the course of the observations, that as the size of the disc decreased, the irradiation of light became greater. The irradiation was so great in Set VI, where the visual angle of the normal stimulus was about 0° 24′, that it was found necessary to put two additional sheets of tissue paper over the open end of the case; and even then the irradiation had a disturbing influence upon the judgments. This increase in the irradiation of light may account in whole or in part for the increase of the mean variation as the magnitude of the stimulus decreases.

Only one set of experiments was made in which the normal stimulus was greater than 1° 20′, and in this set the mean variation was greater than the mean variation in Set II. There has not been a sufficient number of experiments performed to warrant one in coming to any definite conclusion as to the cause of this irregularity. It may, however, be due to the fact that the visual angle is so large that the eye in observing is inclined to make movements which are so great as to interfere with accuracy in judging. Beyond this suggestion nothing further can be stated at present as to the cause of this irregularity.

In examining the Summary Table one will notice a marked correspondence between the results obtained by the two observers. With the exception of Set IV, which was the first made, when the mean variation increases for the one it also increases for the other. Throughout the whole six sets the mean variation for Observer P. has been considerably less than for Observer M.; but on comparison it is found that they bear a comparatively constant relation to each other. In Set I the mean variation for Observer P. is 83.6% of the mean variation for Observer M.; in Set II, 75.1%; in Set III, 84.5%; in Set IV, 81.1%; in Set V, 72.08%, and in Set VI, 93.9%; the average of these is 81.7%.

In addition to what has been said, it is of interest to consider the Constant Error, as seen in the tables. One of the results of Mr. Quantz's experiments was that the movable

disc was always underestimated. This fact, which as yet cannot be accounted for, is decidedly confirmed by these trials. A glance over the last column of the tables, which contains the total averages of the Constant Errors for the different sets, will show that there is, in nearly all cases, a *positive* deviation. This means that, in general, the moved disc was decidedly underestimated.

Nevertheless, on examining the different series of each set separately, it is found that when the variable stimulus is closer to the observer than the normal stimulus, it is judged to be equal to the normal when its visual angle is greater than that of the normal, *i. e.*, it is underestimated. But when the variable stimulus is farther away from the observer than the normal stimulus, it is judged to be equal to the normal when its visual angle is less than that of the normal, *i. e.*, it is overestimated. As may be seen by an examination of the tables, this is constant for both observers throughout almost the whole of the six sets. We may, therefore, conclude, with a considerable degree of certainty, that when we compare the size of two objects lying at different distances, the nearer object is underestimated or the more distant one is overestimated. Although it was the desire in these experiments to take no account of distance and to attend only to the size of the objects observed, it is evident that the observers' knowledge of the distance has had a slight influence on their judgment. Probably the underestimation of the nearer object may be accounted for by the fact that since it is known to be nearer to the eye of the observer than the normal stimulus, it is expected that it will appear larger, and hence it is judged to be smaller than it really is. And in the same way, when the variable is at a greater distance from the observer than the normal, it is expected that it will appear smaller, and hence it is judged to be larger than it really is. It may be mentioned here that these results do not correspond with those obtained by Goetz Martius.[1]

However, the discrepancy may rest upon unforeseen circumstances, which may be ascertained by future research. In the experiments of Goetz Martius, the objects were seen successively, thus making necessary a change of the convergence and accommodation (all of the observations of Goetz Martius were made binocularly, while in these experiments the observations were made monocularly, and one of the objects was always at rest at the same normal distance). In all likelihood the regularity with which the positive and nega-

[1] *Philos. Stud.*, V, p. 601 ff.

tive errors appear in these trials has something to do with the incongruence of the visual angle and the angle of regard; the so-called Parallax of Indirect Vision, to which Kirschmann attributes so great a significance for the monocular depth perception. However, at present the connection is not clearly seen, but the reader, who is interested in the matter, is referred to the articles of this author on "The Parallax of Indirect Vision and the Slit-formed Pupil of the Cat"[1] and "The Metallic Lustre."[2]

It must be remembered that the trials of Goetz Martius, which were concerned with linear magnitudes, had entirely different aims and methods; and, therefore, the results must, to a certain extent, be incomparable with ours. His method was adapted chiefly to the problem whether there is overestimation or underestimation in different distances, and not to find out the accuracy of our judgment for the magnitudes themselves.

How great the accuracy of the judgment for surface magnitudes is may be recognized by regarding the diagrams of Figure 2, in which are given the magnitudes of the discs used as nearly as they can be reproduced in a drawing. The left ones represent the larger, and the right ones represent the smaller of each of the six pairs. (They are arranged concentrically in order to save space.) Before the diagram was drawn it was proposed to represent the accuracy of the judgment, *i. e.*, the mean variation, by the thickness of the stroke representing the circumference; but a simple calculation from the tables will show that the thickness of the stroke would have to vary between about $\frac{1}{6}$ mm. (0.234 for M. and 0.198 for P.) for the greatest circle, and about $\frac{1}{10}$ mm. (0.098 for M. and 0.092 for P.) for the smallest circle. These magnitudes are too small to be well represented in a drawing, and certainly they cannot be represented in a cut. Thus the accuracy of the judgment in these trials on surface magnitudes is too great to be indicated by the thickness of the stroke.

The results of these experiments could be summarized in the following propositions:—

1. The accuracy of our judgment of surface magnitudes is astonishingly great. The mean variation for visual angles of 1° or less was always below 1′ in magnitude, and for angles up to 1° 45′ in magnitude, it never exceeded 1′ 20″.

2. Although the results do not entirely fulfill the demands of the Psychophysical Law, yet they show a certain approximation towards it.

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[1] *Philos. Stud.*, Vol. IX, pp. 447-495.
[2] *Ibid.*, Vol. XI, pp. 147-189.

3. In the comparison of a fixed object with one which is moved towards or from the eye, the latter is overestimated when it is at a greater distance from the eye, and underestimated when it is at a lesser distance. Taking all of the results together, the moved object is decidedly underestimated. This latter fact confirms fully the results of the earlier investigation of Mr. Quantz.

GENESIS OF NUMBER-FORMS.

By D. E. Phillips, Clark University.

Before entering into the results of this inquiry it may be well to make clear the nature of these forms and to give a short account of former investigations. The first is a very difficult task, for psychological phenomena that appear foreign to us are hard to comprehend, especially when so strange. The reader will be greatly assisted in what follows by first examining carefully the drawings given on pages 512 and 513.

The general character of a number-form is such that whenever a number is thought of, it appears in the same place on a visual diagram which is invariably called up, viewed by the mental eye, often definitely located, and which usually consists of an irregular composition of lines on which the figures appear either written or printed. These diagrams are often enormously large in comparison with the drawings here presented. "Sometimes," says Galton, "a form has twists as well as bends, sometimes it is turned upside down, sometimes it plunges into an abyss of immeasurable depth, or rises and disappears in the sky."[1] In some instances the line does not appear; nevertheless, the numbers occur in a fixed order, but are usually less complicated. Galton says the most common way is to see only two or three figures of the diagram at once, but in my investigation, that depends upon whether the mind is performing mental calculations, or the form is viewed as a whole. The entire form can usually be seen, and, by many, as distinctly as if viewed by the natural eye.

"Number-forms," says Galton, "are in each case absolutely unchangeable except through a gradual development in complexity. Their diversity is endless, and the number-forms of different persons are mutually unintelligible. These strange 'visions,' for such they must be called, are extremely vivid in most cases, but almost incredible to the vast majority of mankind, who would set them down as fantastic nonsense;

[1] "Inquiries into Human Faculty," p. 123.

nevertheless, they are familiar parts of the mental furniture of the rest, in whose imaginations they have been unconsciously formed, and where they remain unmodified and unmodifiable by teaching."[1]

The number of individuals possessing such visual schemes depends upon what limitation is put upon the term "Number-Form." For example, some persons have a diagram for the days of the week, days of the month, or months of the year, etc., who have no number diagram. These are evidently of the same nature, and if they be included in our estimate of forms the ratio is much changed. Again we find all degrees of clearness, and some number-forms appear as all other mental imagery, or fade away until classification becomes difficult. So here, as in most fields of investigation, differences on many points result largely from the differences in the extent given to the subject by the various investigators. Surely this will to a great degree account for the variations found in the reports of those who have investigated this subject.

Galton, who published the first article on these forms in *Nature* and afterwards in "Inquiries into Human Faculty," states that "the peculiarity in question is found, roughly speaking, in about one out of every thirty adult males, or every fifteen females."[2] But he considers only clear number-forms and the estimate is on adults. The next work was done by G. T. W. Patrick of the University of Iowa, and appeared in The *Popular Science Monthly*, Feb., 1893. He is inclined to think that one out of every six adults would be a more accurate proportion, that the proportion among children is greater, and that it is perhaps a little more common among women than among men.[3] It is to be observed, however, that among the diagrams which he gives are diagrams for the months, days of the week, seasons, and alphabet; no mention is made of whether they are counted in the estimate of one in six.

The same year Flournoy published his *Des Phénomènes de Synopsie*, in which he includes number-forms. He informs us that he received returns from 370 persons between 18 and 40 years, and found that in childhood colored-hearing or *photisme* is much more frequent and gradually diminishes, while on the other hand schemes are much more stable and endure through life from the time they first exist. At 20, 1 in 6 have colored-hearing; 1 in 9 have visual schemes, and

[1] "Inquiries into Human Faculty," p. 156.
[2] "Inquiries into Human Faculty," p. 119.
[3] *Pop. Science Monthly*, Feb., 1893, p. 506.

1 in 15 have both.[1] About the same time Miss Calkins published in THE AMERICAN JOURNAL OF PSYCHOLOGY her examination of 543 students, all girls, of whom 78 have forms, 32 colored-hearing, and 14 have both.[2] In the fall of the same year she examined 203 new students, finding 32 with colored-hearing, 61, or 30%, with forms, and 17 with both.[3] In the first case about 1 in 7 have forms, and in the last nearly 1 in 3. Here all of the various diagrams mentioned are counted. Out of 300 children, from 10 to 12 years of age, Binet found only 3% with number-forms, but attributed the result to his inability to make them understand what he desired and to their indifference.[4] These variations are largely due to the different limits given to these psychological phenomena, the difference in age and sex of subjects investigated, and the manner of investigation.

The present writer's interest in this subject was first aroused by some strange and complicated answers to the following questions in a syllabus on Number and Mathematics: (*f.*) Cases of number-forms, *e. g.*, the first 12 numbers being habitually associated with a dial or clock-face, the first ten on a line, straight or curved, systems of dots, colors, etc. Do odd seem to you different from even numbers? Draw any number-forms. How do you arrange days of the week or month, the musical scale?" As this section did not cover the points of greatest interest, and as each seemed to have great difficulty in explaining these mental forms in writing, I began a personal examination of 332 Normal School students. Still some questions of importance were overlooked in the early part of the work. The following questions were finally formulated and the investigation extended to 974 school children of Worcester, Mass., and 343 miscellaneous adults personally interrogated: 1. At what age did it appear? 2. How did it originate? 3. Is it useful, or troublesome? 4. Do you see the figures on a line? 5. How large does it appear to be? 6. Where is it located? 7. Are you left-handed? 8. Do you know of any forms in your family? 9. State any peculiarities about your form or its use. 10. Do you like mathematics? 11. Give name, age, and sex.

The following table shows the sources of the material for this article, number of forms collected, the per cent. having some form, and the per cent. having number-forms.

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[1] "*Synopsie*," p. 15.
[2] AMERICAN JOURNAL OF PSYCHOLOGY, Vol. V, p. 265.
[3] *Ibid.*, p. 439.
[4] "*Synopsie*," p. 16.

TABLE I.

SOURCES OF MATERIAL.	No. Examined.	Number-Forms.	Month-Forms.	Week-Forms.	Alphabet-Forms.	Lord's Prayer, Songs, etc.	Total Forms.	Total Having Forms.	Per cent. Having some Form.	Per cent. Having Number-Forms.
Normal School,	332	20	45	13	3	—	81	59	18—	6
Miscellaneous,	343	23	20	8	7	4	62	45	13+	7—
Returns on General Syllabi,	360	25	31	2	4	—	62	50	14—	7
School Children of Worcester, 10-16 y.	974	79	118	11	9	—	217	167	18—	8+
Total,	2009	147	214	34	23	4	422	321	16—	7+
Males,	969	67	74	19	13	4	177	145	15—	7—
Females,	1040	80	140	15	10	—	245	176	17+	8—

The 332 Normal students I examined personally, and had opportunity to see some of them a year later, but found no change of any note. Fully 80% of this class are teachers. The 343 miscellaneous are also cases personally investigated. Nearly all of them are adults over 25 years old; 92 are teachers; 41 Clark University men; the remaining persons are, generally speaking, uneducated. A few cases were obtained by correspondence. Many answering the syllabi omitted points of importance, and some not included here simply spoke of the direction numbers take in their minds. The returns from the Worcester schools were collected from the 7th, 8th and 9th grades, from children of 10 to 16 years old. After a short explanation the children were asked to draw whatever form, or forms, they had. In all the rooms, except five, an effort was made to keep them from obtaining an imaginary form for the occasion, and in only a few cases was there reason to think the forms were not genuine; each pupil giving a form was, as a rule, questioned privately. In

the five rooms fair opportunity to copy or devise a form was offered pupils by presenting drawings and entering into detailed explanation; but the forms collected there show no signs of fraud. The percentage of forms was not quite as large in three rooms as is usually found. After giving the slightest explanation, a close observer will hardly fail to distinguish every one having distinct number-forms. Those who have no form have no idea of what you are speaking of, and are often slow to comprehend any explanation, appear surprised or treat indifferently what you say. Those having a form show an entirely different attitude. In a room of 44 children the mere mention of forms showed four who comprehended my meaning, and they were afterwards found to be the only ones having forms. Six men in a shop were asked if any of them had a number-form. One asked, "What do you mean?" On a word of explanation he exclaimed, "I have the craziest thing you ever saw," and proceeded to outline No. 12, Plate II. While at their play I asked two girls, about five or six years old, if they could count one hundred. Each replied that she could. When asked which way the numbers go, one replied, "They go round and round, then up to the clouds," making a circling motion with her hand. In the first grade I examined 23 children privately, finding two clear number-forms. The first boy was a little over six and had just learned to read. While he was counting 100 I observed that at certain points there was considerable struggle, as if he had lost sight of something. When asked if the numbers went down, he shyly said, "They go up and turn," and then drew on the board Form 17, Plate I. I tried to deceive him by placing numbers on the turns to suit myself; five different numbers were placed where 17 is, but each time he shook his head; finally he told me to put 17. He located all the others with the same certainty. Later the form was slightly changed; the boy was called and asked if it were correct. He is very shy and slow, but soon proceeded to correct it even to the length of the lines, which he was not tall enough to draw himself. The clearness and persistency of such a form are not to be doubted.

Some will be inclined to mistrust results obtained from children from 10 to 16 years, and while every precaution was taken to avoid error, I do not consider that this part of the study is free from such. But it cannot be far wrong, for almost the same ratio exists among adults, none of whom remembers a time when his form did not exist. The ratio, however, is, as we shall see, of little importance, and the culminating point of this work lacks no reliable evidence.

A few things in Table I are worthy of attention. The ratio

for number-forms remains much more constant than for other forms, while children have more week and month forms than adults. The total number of forms is nearly one-third larger than the number of persons having forms. This shows that several have more than one form of some kind. Where a distinct number-form is found, the individual often has other forms, but many that have a month-form, week-form, etc., have no number-form. The last two columns show that the per cent. having some form is more than twice as large as for those having only number-forms, that the per cent. decreases a little with age, being about 1 in 12 for children and about 1 in 15 for adults. The proportion seems to be a little greater among females than among males; especially is this true for month-forms. Omitting these there is by no means so great a difference as Galton found. As a rule men think such things very trivial and are not as willing to respond as women. As we hope to show later, unless an arbitrary definition and limit be given to forms, a table of this kind can be no accurate guide. The records here are based upon comparatively distinct forms. Since completing this table several returns have been received that might change the ratios somewhat. Prof. Barnes, Fairmount Normal, W. Va., states that on examining 118 boys and 136 girls, he found that 69 boys and 53 girls had some form. But there was no personal examination and no forms drawn.

TABLE II.

General Direction, First Turns, and Endings.

	No. of Forms.	TURN.					END.		
		To Left.	To Right.	At 10	At 12	At 20	Below 100	At 100	Above 100
Galton,	65	23	38	15	27	5	16	26	22
Flournoy,	37	4	29	14	5	5	10	14	12
Patrick,	14	2	9	2	0	0	5	1	5
Present Collection,	147	18	91	60	32	11	78	53	16
Total,	263	47	167	91	64	21	109	94	55

Table II gives a classification of the principal direction,

Plate 1.

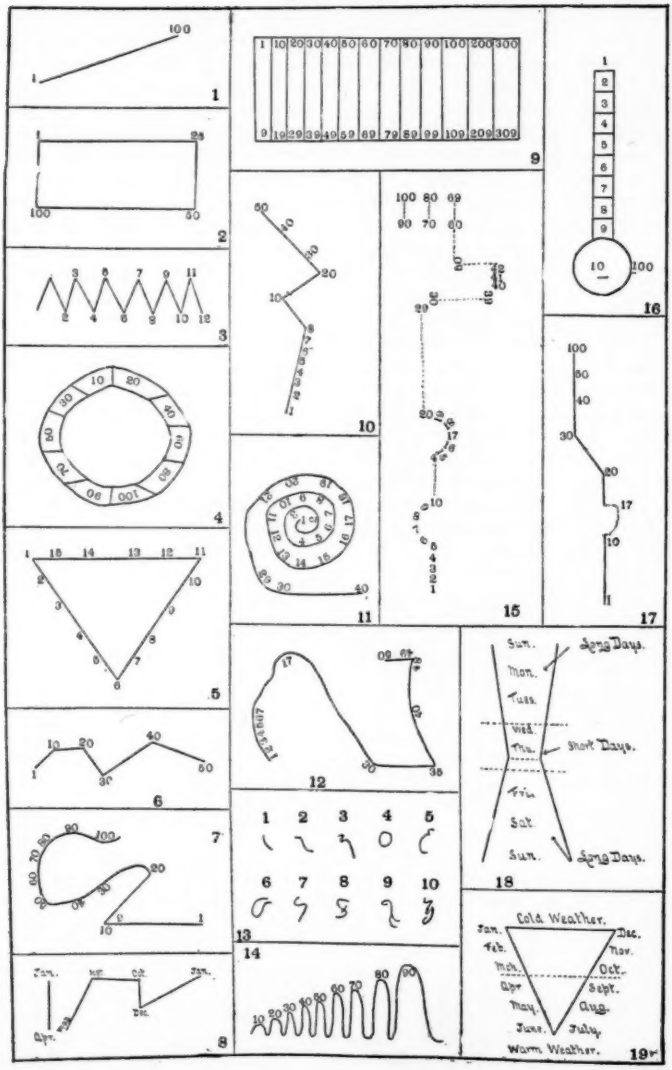
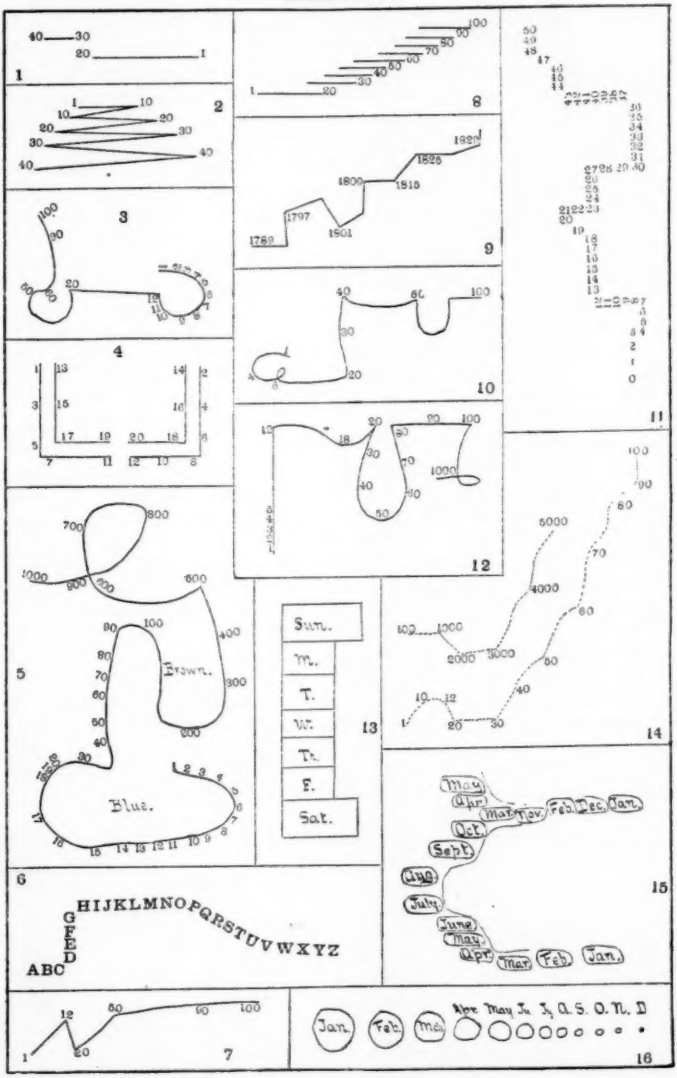

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
Sun.
Mon.
Tues.
Wed.
Thu.
Fri.
Sat.
Long Days.
Short Days.
Cold Weather.
Warm Weather.
Jan.
Feb.
Mch.
Apr.
May.
June.
July.
Aug.
Sept.
Oct.
Nov.
Dec.

Plate 2.


1
2
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5
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10
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12
13
14
15
16
Sun.
M.
T.
W.
Th.
F.
Sat.
Brown.
Blue.
ABC
HIJKLMNOPQRSTUVWXYZ
May
Apr.
Mar.
Nov.
Feb.
Dec.
Jan.
Oct.
Sept.
Aug.
July
June
May.
Apr.
Mar.
Feb.
Jan.
Jan.
Feb.
Mch.
Apr
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Ju
J
A.
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O.
N.
D

first turn, and ending, of all the number-forms I have been able to find published, including my 147. The direction and first turn have been referred to by others, but no definite summary made except that Miss Calkins directs attention to an outline of turns. Her work is not included here because information on other points was wanting and few forms were published. Out of 68 forms she gives 17 turning at 10, 8 at 12, and 5 at 20.[1] The above table is but a rough classification, but it shows some tendencies in number-forms for which the psychologist may be puzzled to find an explanation. A glance at the total directions and turns shows that these do not include all the forms, for a numeral scheme may take any direction and make its first turn at *any* number, but these show such majority as to point to some general laws. Very few forms go down, but some are so complicated in direction that classification is impossible. The endings are only approximately correct, for many do not draw the entire form, and some forms are capable of almost indefinite extension according to the series in mind. We shall have occasion to refer to the lines here indicated later.

Plate I shows the leading types and some of the most highly developed forms of persons under eighteen; Plate II contains only adult forms. These two plates indicate that number-forms develop in complexity, and perhaps change somewhat in shape, notwithstanding that most individuals think that their form has always been substantially the same. This contrast and the three leading types, to be mentioned presently, would appear more striking if the entire collection could be published.

It seems almost impossible to classify these number-diagrams. Such a classification is perhaps of not very much value, and must be based upon other points as well as on the drawings. The variety and complexity of forms are wonderful. Month-forms are frequently alike, but I have found no two number-forms exactly the same. Still, taking several points into consideration, we may note three rough types which include nearly all number-forms. Nos. 1, 2, 5, 9 and 16, Plate I, and Nos. 1, 4, 8, Plate II, represent the simplest that it seemed advisable to include, although there is really no line of demarcation. The chief characteristic of this class is the evidence of conscious construction, or of deduction from, or alteration of, some form or object more or less frequently before the eye. Of 19 who have suggested an orgin for their number-form, 15 belong to this class. As regards localization, vividness, etc., these individuals are, as

[1] AMERICAN JOURNAL OF PSYCHOLOGY, Vol. V, p. 449.

a rule, most uncertain. The fact that they frequently locate the forms on blackboards, charts, books, blocks, etc., is evidence of such an origin, and of the fact that they are slightly modified mental images. One lady says she is conscious of taking hers from blocks on which the numbers were stamped; and she is the only one who has complained of a form being troublesome. Another is conscious of arranging the numbers on the edge of a chart and ever afterward having the form. This will explain many that take a real geometrical form. As a rule this class does not exist to any great extent among adults, and since adult forms can hardly be modifications of these, it is probable that they drop out early in life and that those formed most unconsciously are most persistent. They are never so striking to the individual nor used with half as much freedom and elasticity as those that seem more spontaneous and to others more unmanageable.

Nos. 3, 6, 8, 10, 15 and 17, Plate I, and Nos. 2, 7, 9, 11 and 14, Plate II, furnish examples of the largest class of forms. They are usually pronounced useful, but not so vivid as the more complex ones. To this belongs Diamandi's mentioned later. More than half the forms collected belong to this class, and the similarity between them is sometimes considerable.

Nos. 7, 11 and 12, Plate I, and Nos. 3, 5, 10 and 12, Plate II, show a decided tendency to take a circular or curved direction. They are usually exceedingly vivid, strangely located, and generally said to be useful in a manner and to a degree wholly unintelligible to any one except the possessor. There are but few of these and they usually extend beyond 100.

Besides these three classes there are some very exceptional forms. C. C. S. and L. B. have forms of three dimensions. Mr. L. of Clark University tried several times to draw his, but could not. He finally described it as resembling the threads of a screw about four feet in diameter, the threads enlarging on the far side, and after the first round running almost parallel. The first one hundred completes a round, and each succeeding hundred a little more than a round, ending directly over 2, 3, 4, 5, etc., respectively. If two thousand is thought of as twenty hundred, it appears on its proper round over twenty, but if as two thousand, it is located on the same round, but over two. The form as a whole is located on a plain in front, goes to the left, and sometimes seems to extend behind him. It is very definite and clear, and highly useful in all numerical calculations. No. 13, Plate I, was obtained from a Swede boy of 17. I

could not understand the peculiar characters under the numerals; two personal interviews gave no information except that these are directions which the numbers take when he hears them, and that the same feeling is received when he sees them, unless they are passed over rapidly. The boy is timid and slow to learn. The characters are reduced to about one-eighth of their original size. Later a girl of fifteen presented a similar case. Her characters do not resemble those of the boy, and are somewhat larger. She gives the same explanation, except the power to receive such a sensation from sight is gradually diminishing. She is an American girl of average intelligence.[1] These two cases are peculiar, and call to mind an observation made some ten years ago, on a boy about 12, in my school, who said that each note had a peculiar swing to him when spoken, and when singing he had an impulse to follow the same.

Let us now examine the answers to the questions asked. Of 321 giving forms, 280 answered the question: "At what age did it appear?" 241 do not remember a time when it did not exist; 17 feel sure that their number-form originated about the age of 6; 22 place their month-form after 12 years; 11 say they learned to add and multiply on their scheme. Prof. Patrick gives quite an extensive form of a girl 9 years old; No. 7, Plate I, is from a girl only 6, and No. 17 has already been mentioned. Nothing is more marked than the very early period at which these schemes are developed. Probably number-forms begin with the naming of numbers, and go on as the child learns to count one hundred. There are several things in the forms that point to this fact. Most children can count one hundred before they learn to recognize anything written or printed. This begins at a period when, of all others, the largest part of our knowledge comes through sight; it is only natural that they should try to visualize the abstract and to cast it into some concrete form or space relation. Table II shows that of 263 forms 91 make their first turn at 10; 64 at 12, and only 55 extend beyond 100. There is at least a suggestion that both the first turning points and the limits of these forms may be related to the manner of counting, and indicate that their formation begins with the earliest counting. A child is generally taught to count 10 on its fingers, yet this is quite different from its previous knowledge, which chiefly consists in learning the names of things. It by no means recognizes the numbers as the names of its fingers. Generally there is

[1] Cf. the somewhat similar diagrams given by Miss Calkins,—AMERICAN JOURNAL OF PSYCHOLOGY, Vol. V, Fig. 7, Plate I, facing p. 464.

a pause in the counting process when ten or a dozen is reached, then it is again taken up only to plunge the child into pure abstraction, and continued until 100 is reached. A friend writes, "F., 4 years old, does not know his letters, but he can count 100." Learning to count before learning characters of any kind is the rule, almost without exception, among country children. Of 1160 individuals, including 660 children, 280 high school students, and 220 adults, 92% learned to count first; over 80% assert that they could count 100 before they could read.

In this connection I wish to add some testimony on a point that will doubtless seem mystical and incomprehensible, but I cannot forbear to state the facts. When my work was presented at the seminary, the question was raised whether these forms could precede the power to recognize the written or printed figures. To my astonishment two possessing forms immediately declared that they were confident that their forms existed before they knew written or printed characters of any kind. After this I talked the subject over with Dr. Story, Professor of Mathematics at Clark University, who thought this impossible, and attributed it to their inability to remember accurately the period of absence of a thing so long and continuously in the stream of consciousness. However, since that time I have reached as many persons having forms as I could conveniently, and many who profess an early recollection of their forms give the same testimony. Besides this there are cases of individuals having forms for certain anthems, the Lord's Prayer, and Doxology. These are not mental images of a printed page, but a projection of an indiscernible something into space. Dr. S. sees the first phrases of the Lord's Prayer nearest; the remainder gradually recedes. This was learned early, while the creed, learned from a book, is reversed. In a like manner early number-forms do not give a visual image of any characters at first, but, so far as I can understand, simply a division of space in these pecular directions. Mr. T. states that his form existed as early as the fifth or sixth year, yet he did not know figures and was unable to read until he was ten. Mathematical prodigies have exhibited a great power to visualize numbers, yet in most cases the power was well developed before learning to recognize printed characters. It is here, as we hope to show later, that the best approach to the genesis of number-forms is to be found.

As already stated only 19 offered any explanation of the origin of their numeral scheme, and in most cases the explanation is not satisfactory even to the giver. Twenty-four give an explanation of their month-form. "Brother, 5 years

old, cannot add when not in a room with a clock. He counted by the hour spaces; he is now 7, and uses the clock face mentally." Lady of 26 years says, "I learned to count when 5 years old, but could not remember 11 and 12. My grandfather told me to count two 6's, and that made a dozen. For some time I calculated time and objects by 6's; never how many 12's, but so many clock-faces. I used to arrange pebbles in the sand as on a dial. I can't explain to anyone how I can count faster that way, but I can." Such a description may safely be trusted.

Is it useful, or troublesome? To this point particular attention was given because some investigators have considered such forms a hindrance to abstract thought. But only those who have no diagram make such an objection.

Mrs. Hornbrook has given in the *Educational Review* a review of some work in which she attempted to show the utility of number-forms. She gives there her own form, which has been of so much service to her. The numbers up to 100 were arranged in the form of a square. In some tables the 10's were put in very heavy type; in others the multiples of 5, 6, 8, 9, etc. These tables were always kept where they could be seen, with the hope of producing a form in the child's mind. She states that they were not able to discover any permanent forms, but that forms were developed and used while learning the multiplication table.[1] I am inclined to think that such forms were only localized memory of the tables, such as anyone can form, by a voluntary effort, of objects often seen, and doubtless closely related to the first division of forms already given. As to the utility of number-forms in many cases, I have no doubt. Of those having some form 211 answered this; 97 are sure of its utility; only one finds it troublesome; 113 say it is neither useful nor troublesome. From many I receive such replies as, "very useful," "could not add without it," "use it every day," "yes, I shut my eyes and count." Mr. B. says he cannot see how mathematical calculations can be performed without such. A girl of 18 states, "I could not add 7 and 9 without mine." Another, whose form appears on Plate I, No. 6, informed me that she put herself to sleep counting on it. Girl, 20: "In rapid counting I know at every notch on the line just what figure belongs there." "My form," says Mr. L., "is as necessary as bread and meat." In Miss Calkins' analysis of this point, about one-half do not think them useful, but she mentions no cases where they are considered detrimental. Now, those who are neutral are doubt-

[1] Vol. V, p. 479.

less more nearly correct. They perhaps use their form just as we use language, without ever thinking that it is useful as a medium of thought. In fact, several who have said they did not use their form, have later corrected this statement by saying that they found themselves continually using it. In using this complex machine they always speak of "jumping" from place to place. The best way to understand this is to look at a surface, and then fixate the eye first on one point, then on another. This doubtless explains why in calculating many see only parts of the diagram. Fig. 3, Plate II, is a carpenter's. In telling how he used it he said: "When I reach 20 the horizontal parallel lines turn upright; the space between 10's is divided into equal parts, except the middle is larger for 5. If 9 were to be added to 37, which is now written in its place, I jump to 46."

With the exception of Diamandi, who sees his on the convolutions of his brain, the rule is to locate in space near the head, when located at all. Several did not answer this question, and many were not conscious of any localization; but the most common replies were, "right in front of my eyes," "it is not located, but just stands out by itself," "I can see it hanging in the air right in front of me," "it always goes to the ceiling," "I can see it on the wall." Four localized their forms to the left about three feet, three overhead; some said it went up in front until they could hardly see it. When drawing his form one man found an ordinary sheet of paper too small, saying, "It ought to lengthen out," giving his hand a swing to the right. Flournoy gives the following cases: "M. Y.'s number-form, composed of parallel lines representing the hundreds, occupies the right half of the space in front of him. In the left half floats his diagram of the week in the form of a horizontal rectangular figure divided into seven bands, something like a leaf of ruled paper, floating in the air about a metre from him, opposite his left thigh. Still more to the left, and at the height of his head, is situated his year-form, an ellipse of small eccentricity presented in nearly a vertical plane. Whenever M. Y. thinks of a date of the year, of a day past or future of the current week, or of a number, he perceives it in its proper place on the corresponding scheme. I have often had occasion to make him write down rapidly a series of figures at random. He follows what he is writing only with an indirect vision, like a hurried copyist who lets his hand work of itself, and will not lose sight of the page he is copying."[1] He gives also an account of a girl, 11 years old, who had a diagram for

[1] "Society for Psychical Research," Vol. VIII, p. 146.

the months situated directly over her head, entirely out of the field of vision, so high that she could not reach it by 6 to 8 centimeters. It always kept the same distance from her head. Another sees the future in front and the past behind him. In order to see into the past he must place himself back in his imagination beyond the event, saying it is impossible to visualize time which is behind him.[1]

The space in the forms between the turns after 20 are not usually filled out, but the numbers when thought of take their places in order. As a rule near the end the numbers seem to be very much crowded, but those who describe their form as extending up into the sky,or far away in the dim distance, have no such trouble. In some no line appears; in others the line followed is a dotted line which the individual is conscious of tracing; but in the majority the line is somewhat distinct.

From Table II it is evident that about four times as many go to the right as to the left. Having found two left-handed persons with forms running to the left, it was suggested that this might have something to do with the direction. Miss Calkins kindly re-examined her subjects for me on this point, but found nothing to substantiate it. In my further investigations I have met with the same result. So the direction must have some other explanation.

Galton thinks these forms hereditary, with even sometimes a strong tendency to reproduce the form.[2] But as strong a similarity of forms is seen in many not in any wise related. An examination shows two or three general types, no matter where they are collected. Miss Calkins' result is much in favor of heredity. Flournoy says all that can possibly be attributed to heredity is a *predisposition*.[3] My answers on this question were, children not included, 128. Six were sure of relatives having a number-form; 17, of relatives having other diagrams. I find no more signs of, nor reasons to look for, heredity here than in any highly developed power of imagination, memory, art, music, etc., all of which are much questioned since Weismann's theory of heredity has become prominent. Two in the same family may have like forms by mere coincidence.

Ninety-four of those having number-forms assert that they are fond of mathematics. "I worked at odd times," says one, "for two years on a geometrical puzzle, and finally got it." "I like mathematics," says another, "but think my

[1] "*Synopsie*," p. 183.
[2] "Inquiries into Human Faculty," p. 140.
[3] "*Synopsie*," p. 203.

form has nothing to do with it." "I love to solve problems mentally," is a common reply. Twenty-eight "hate" mathematics. There are so many things to modify one's likes or dislikes for mathematics that it seems to me this proves nothing one way or the other. In my work I have tried to ascertain whether they are more general among dull persons, or the more intellectually active, but so far I have been unable to detect a stronger tendency in the one class than in the other. Neither does it appear that they are more general with the imaginative. This differs from Galton's statement that "it is found among most imaginative persons."[1] More number-forms have been found among forty-one persons in Clark University than in any equal number elsewhere, eight well defined forms having been found. There are three of three dimensions. My observation among higher mathematicians has been too limited to draw any conclusions. No one can say that one would be better without a form, for, as it appears only when numbers are thought of, it does not influence the ordinary thought processes.

In this connection it will be interesting to *résumé* the comparison which Binet makes between Inaudi and Diamandi, two mathematical prodigies.[2] Inaudi was born in 1867 of a poor family. During pregnancy his mother frequently saw the recklessness of her husband, and, pressed by poverty, she calculated in her head means of economy; her days were spent with figures until she acquired a mania for counting. This is the statement of his brother. At the age of six Inaudi acquired a passion for counting; never used fingers, pebbles, etc., but did all with words, learned the names to 100 from his brother and then demanded more. He entered Paris at thirteen; could neither read nor write. He was presented at the Academy of Sciences in 1892, and in 1892-3 was examined more than fifty times. When an infant his head was so large that it was thought that he could not live. He learned to read seven years ago, converses but little, yet shows good natural intelligence. In calculation or reproduction of figures he repeats the numbers three times, seems perfectly quiet, and can carry on a conversation on other subjects, the only effect being to prolong the time. Binet thinks that he must have over 300 figures in his mind at the same time.

Diamandi was born in Greece in 1868; entered school at seven; was at all times first in mathematics; left school at sixteen; became a grain merchant, and here his mathematical talent greatly developed. He had fourteen brothers and

[1] "Inquiries into Human Faculty," p. 114.
[2] "*Psychologie des grands calculateurs et joueurs d'échecs.*"

sisters; only one brother and one sister possessed a similar aptitude. His mother had a wonderful memory for everything. He has abandoned commerce; reads much; everything written on mental calculations; composes verse and romances, and knows five languages. In 1893 he presented himself at the Academy of Sciences, and was examined by the same commission that examined Inaudi. Binet has experimented with him fifty times, from three to five hours each time. He came to contest with Inaudi, and it is this that makes the account of Inaudi of value here. Inaudi is an exception among mathematical prodigies. He is, so far as understood, of the purely auditory type, while Diamandi is of the visual type. For some time Diamandi denied having a number-form and kept it concealed for more than two months. Once he said that the figures appeared to him on one of his cerebral convolutions placed to the front and left.

"The form and location of this image in relation to the individual, are the elements which volition can scarcely modify." His number-form is of the usual type, extending from left to right, of broken lines, and space more occupied at the beginning. Besides this, Diamandi sees all objects in the centre of a complex figure formed by a grayish-colored mass, enclosing a lighter spot. The thought of a house, a dog, etc., brings the image of such into the lighter space. He first presented to Binet a roll of paper with 2,000 figures on it, any one of which he could reproduce and locate without difficulty, or read them diagonally or otherwise. When hearing figures in French he encounters great difficulty, being obliged to translate them into his native Greek, and often makes many errors, but never fails when they are presented on paper. After a first look at the figures on paper, he closes his fists; puts them against his temple; bows his head; soon takes another look at the paper, then closes his eyes and begins the operation. With him the making of a visual image is the important thing, while Inaudi appears to make no mental representation. Diamandi desires the figures written in a square, and he always begins at the left, no matter how they are written. The multiplication of 65,879x2537 was accomplished in 3 min. 10 sec.

If interrupted by noise or questions, Diamandi loses the image then in mind and is obliged to reproduce it. No matter how many of the figures are variously colored, he reproduces them with their proper color. With a small number of figures Inaudi is more rapid than Diamandi, otherwise Diamandi surpasses him in rapidity and in extent of reproduction and calculation. Whether this difference is due to other causes than the existence of Diamandi's num-

ber-form, Binet does not consider; although it is certain that he in some way makes use of it, always locating the figures in their proper places in his form in a manner which only those who have such diagrams can comprehend. Then they are as plain as if written before the natural eye. I give these two cases here because of their bearing on forms and on mathematical prodigies in general. Unfortunately number-forms did not gain attention early enough to be studied in former mathematical prodigies. A few are known to have possessed true number-forms. Dr. Scripture says the great peculiarity of mathematical prodigies is the visual images of numbers which they always carry about in their minds. And among general returns there is mention of three children extraordinarily rapid in use of numbers and each has a well defined form.

Flournoy *résumés* several thories which have been held as to the explanation of the whole matter of *Synopsie* as follows: All are analyzable into two great classes. The first seeks the explanation in psychological association. The other declares it outside of ordinary association and seeks its explanation in physiological conditions, holding that the continuity of the central cortex permits excitations to radiate to different centres of localization. It is an exceptional anastomosis uniting nerve fibres or cells ordinarily separated. Flournoy holds that the principle of psychophysic parallelism will harmonize both theories. He classes all the phenomena of *Synopsie* under affective, habitual, and privileged psychical associations. Affective association is the general coloring which each sensation gives to every other sensation received at the same time, no matter how heterogeneous. This is only natural from the unity of the nervous system. Habitual association is the continual association of two things until they become an indissoluble whole, such as months and days in columns of an almanac, etc. Privileged association plays an enormous rôle. Things become indissolubly fixed in our memory and thought, not because they are often in the field of reality, but because of a time, perhaps an only time, when the thing struck us and left an indelible trace in our nervous tissue. Such are the visions of our early childhood which have submerged the other memories of the same period, and no reason can be assigned why such remained in preference to a thousand other scenes. Krohn gives quite an extensive review of the theories concerning Pseudo-Chromesthesia, or colored-hearing. He claims that none of the many physical explanations nor the psychical association is comprehensive enough to explain all of the facts. However, we infer that he does not consider number-forms a closely allied

phenomenon, styling them *automatic* associations. He considers such explainable by psychic associations.[1]

Having presented the main body of facts, we may well ask, what does all this mean? Have we an exceptional phenomenon, unrelated to any other psychic activity, for which some special explanation must be given? Concerning the general explanations of *Synopsie* above presented, we have little to say, but it appears certain that these visual diagrams are only less ordinary examples from a much wider field of mental phenomena.

One of the most striking things about number-forms is their extremely early origin, yet that they have a period of gradual formation and development is also evident. Forms for the Greek alphabet, historic dates, the years that one has lived, months, and in a few cases, numbers, have been developed late in life. As a rule they are not so vivid and enduring, but there are striking exceptions. Mr. G. says, "My form resulted from an evening reverie while looking in the fire; and after studying higher mathematics, I added minus and plus infinity. I was about 14 when it originated." We must then conclude that forms may originate late in life, and that they certainly become more elaborate by use and time.

There is no more reason for isolating these mental activities from a much larger field than there is for isolating exceptional cases of memory or imagination from these general powers of the mind. In any new line of investigation, the exceptional always receives attention first. This seems to have been the case with Galton, and most of the others mentioned have followed largely in the same path. Beside the more complicated cases which we have been treating, we find more than as many persons in whose minds the numbers simply take a distinct direction. Sometimes the numbers are seen on a straight line running only to the right, left, or up, etc. Sometimes only the numbers appear. Again there is only the sensation of following in some particular direction whenever numbers are thought of; sometimes only a feeling that large numbers are far off. Prof. Story, in conversation with the writer, denied that he had a number-form, but finally remarked that large numbers appeared far off, although he saw nothing, and had no feeling of any particular direction; yet if 7 and 69 were thought of, there would appear to be room between for the other numbers. I felt sure that investigation on this point would throw light upon the subject, and at once began a re-examination of those who had denied

[1] AMERICAN JOUR. PSY., Vol. V, p. 20.

having a form, and others who thought they had none. Two hundred and fifty adults have been reached, and two hundred and ten have a feeling that numbers in some way recede from them. Many report that they have an upward movement. Mr. F. said he could not resist thinking of numbers as going up, the large ones getting very high. With others they appear to go straight in front, or at angle of 45 degrees. To many, large numbers simply appear to be far away, and they are unable to designate any direction. Mr. B., a man of mathematical ability, said he had no number-form, but his number series contracted. Beginning with zero it is more than twice as far to 100 as it is from 100 to 200, and so on towards infinity. This is exactly the law that can be seen in every number-form, though not in any fixed ratio. I believe *that nearly all persons possess some idea of extension of numbers, more or less indefinite.* At first I had no idea that any similar phenomenon could be found in my mental activity, but when I think of 99 in its relation to 1, the form appears about two yards in front of me at an angle of about 45°; and I have never been able by an effort to think of it otherwise except for a moment. I find that but few have even noticed how they cast numerical thought into space relations. Out of 480, above the 7th grade, 74% visualize all mental operations with numbers in some way; and but few had thought it could be otherwise. The suggestions in former studies led us to an examination of children. In the 1st, 2nd and 3rd grades 785 children were asked to count and then to tell if the numbers take any direction. The most unexpected directions were asked for first. Thirty-four said they went down; 60, to the right; 29, to the left; 74, right forward; 470, some upward movement. Of course none of these were considered with the number-forms, but we evidently have the same material here and many of these are probably as distinct as the complex forms.[1]

Here we see the dominance of the eye over the other senses. In most of the forms it appears that there is a tendency to keep within the field of vision; only in a few cases are the forms located outside of that field, and many seem to turn in order to avoid getting out of it.

The number series is a succession, the rudimentary concept of which dates back further than the actual numeration of objects. When children begin to count they usually represent this series by nods or movements of some kind. Many adults unconsciously make these movements. A girl, 17,

[1] It would be extremely interesting to know whether those blind from birth make use of any such space imagery.

always moves her great toe. She says it is irresistible. If these movements are suppressed we might still find them represented in thought in the form of a space series. Without doubt children tend to connect some movement or extension in space with numbers, and it is here that we are to find the genesis of number-forms. Back of any visual image seen by the mind's eye is the motor element in thought, which must have space as a back-ground. Some say they cannot think of any series of objects, as man, dog, cow, horse, etc., without some idea of succession in space. Infinity usually suggests a never-ending line, the direction of which is often definitely located. Many, in thinking of the distance between two places as so many miles, see it in space.

Can early association explain this general tendency to cast the number series into spatial form? Is there anything inherent in the number concept which in any way determines the association? What is the relation of geometry and numbers? Numbers are generally defined as a series of successions, and a number is said to have no relations except that it comes after, and is followed by another, while a geometrical figure may have several relations.

Euclid did not make that wide separation of geometry and arithmetic now so commonly practiced, but there is a general tendency to base primary mathematics again more and more on geometry. Sylvester said, "Every time I go deep enough I find a geometrical bottom."[1] Again we may ask: Are the facts herein set forth to be traced to the general fusion of sensations received through different senses? If so, then this subject is a part of a still larger one, including Pseudo-Chromesthesia, dramatization of numbers (that is, the giving to certain numbers the characteristics of certain persons, animals, or objects), and the whole range of facts which Wundt includes in *complicative association*. The more we attempt to trace any set of sensations or perceptions to their ultimate origin, the more this confusion or mingling is encountered. But it hardly seems best to class these phenomena under the same head. It is better to seek the explanation in the motor and space elements in thought. If I have succeeded in showing that number-forms and others are not such isolated phenomena as they have been considered, and that they have their genesis in simpler mental activity, the object of this short paper is accomplished. Local relationship is very dominant in all thought, and when we suppose we think abstractly, as a rule, we use, either consciously or unconsciously, some substitute. Such thoughts are at bottom symbolic, and in so far concrete.

[1] "Address before the British Association."

I desire to acknowledge my indebtedness to Drs. Hall, Sanford, Burnham and Story for suggestions, to the authorities of the Worcester schools, to Supt. Carroll and the teachers of Worcester for coöperation, and to Profs. Luckey, Barnes and Deahl for the sending in of valuable material.

THE PSYCHO-PHYSIOLOGY OF THE MORAL IMPERATIVE.

By James H. Leuba, Ph. D.

Among the many experiences of the ethico-religious consciousness, there is one of such exalted mien and striking distinctness, announcing so loftily a mysterious will and playing such a remarkable rôle in the higher life of man, that it has long since been singled out by name among ethical races. Kant has baptized it: "*the Categorical Imperative.*" It would be difficult to overestimate either the practical or the theoretical rôle played by this ever-ready monitor. Theologians and naturalists alike have recognized in it the highest and most distinctive endowment of man; "I fully subscribe to the judgment of those writers who maintain that, of all the differences between man and the lower animals, the moral sense or conscience is by far the most important," says Darwin, in the chapter on the Moral Sense in the "Descent of Man."

It is after this inward pattern that the ethico-religious seers and prophets have drawn the features of the objective universal Moral Order. In the past no attempt has been made to analyze the Moral Imperative[1]; it has been considered a simple, ultimate fact and it has moreover been supposed to be, somehow, of another order than the rest of the psychic life. Currently, it has stood for the "divine" in man. Kant himself accepted the fact uncritically as the inscrutable expression of 'universal reason,' as "the absolute dictator of its own laws," and bestowed upon it sundry distinguished appellations, such as "it is an *a priori* synthetic-practical proposition." He was not anxious to discover the *deus ex machina*, but rather, conformably with his general psychological dualism of sense and reason, to make out by repeated affirmations and by appeals to experience an independence of the Categorical Imperative "of any particular tendency proper to human reason, and which need not necessarily hold for the will of every rational being."[2] "From what has

[1] For a summary and discussion of the theories of the Nature and Origin of Conscience, see Chap VI and VII of Hyslop's "Elements of Ethics."

[2] These quotations are taken from the "Fundamental Principles of the Metaphysics of Morals," tr. by Theo. Kingsmill Abbott.

been said it is clear that all moral conceptions have their seat and origin completely *a priori* in the reason. . . . ."

Although to-day the old-fashioned dualism of sense and reason has been set aside in the higher scientific circles, and although psycho-physiological science is now in condition to provide the necessary data for a detailed psycho-physiology of the Moral Imperative, the men of acknowledged authority have not directly addressed themselves to the consideration of this problem, and the superannuated Kantian metaphysical psychology of ethics has not yet been formally superseded by a psycho-physiology of the Moral Imperative in harmony with modern science. Unfortunately for ethics, it has remained chiefly in the hands of those who were not able to launch it in the new current; it still continues to waste its energy in the quest of the *summum bonum*, of the *criterion of conduct*. Whether this criterion be conceived of as a sort of Platonic archetype, the vision of which is to serve as a pattern of right living, or whether it is untranscendentalized and derived in Aristotelian fashion from a "thorough-going and exhaustive cross-examination of men's actual moral judgments," as if it was the centre of the complete circle formed by these judgments,[1] it is, in our opinion, an equally profitless chase with regard to practical ethics: profitless in the first instance because our cosmological concepts do not allow us to believe that there are such archetypes to be discovered; profitless in the second, though theoretically attainable, because it can not be the "ultimate standard" of our judgments about conduct, inasmuch as life is not best represented by a circle, but by a spiral or some such curve, expressive of the guiding modern belief in evolution, in growth. That opinions, such as the one quoted, should still be entertained and acted upon by some of our younger moralists, is a regrettable proof that the evolutionary view of life, if theoretically accepted, has not yet leavened our mental habits thoroughly enough to guide our thinking efficaciously.

Evolutionary ethics has, in its way, accounted for the genesis of the feeling of obligation. But as its method is not psycho-physiological, but rather historical, it cannot be expected to furnish us with the psycho-physiology of the Moral Imperative. When Spencer writes in the broad, undifferentiating manner, characteristic of first attempts in a new field: "We see that where the consciousness of authority, of coercion, and of public opinion, combined in different pro-

[1] James Seth: "The Standpoint and Method of Ethics," *Phil. Review*, Vol. VI, No. 3.

portions, result in an idea and a feeling of obligation, we must class these as ethical, irrespective of the kind of action to which they refer,"[1] we answer, yes, it is probably as you say, but, as psycho-physiologists, we should like to know more about that "consciousness of authority;" we should also like to understand better how the combination of certain consciousnesses brings about a feeling of obligation, and the well-deserved criticism by Prof. Dewey comes to our mind: "Their great defect is that they do not give us any method of differentiating moral coercion (or obligation) from the action of mere superior physical force." "The theories [of Bain and Spencer] must logically commit us to the doctrine that 'might makes right' in its boldest form."[2]

In the following essay we attempt to demonstrate the psychological parentage of the Moral Imperative and its physiological mechanism. We shall be led to set down the thesis that *the Moral Imperative is the psychic correlate of a reflective, cerebro-spinal, ideo-motor process, the efferent end of which is organized into motor tracts coördinated for a specific action.* And we shall endeavor to show how the particular qualities of this experience are—on the generally accepted principles of psycho-physiology—satisfactorily accounted for by this physiological mechanism. In closing we shall direct the reader's attention to the most important practical deductions to be drawn from the advocated conception.

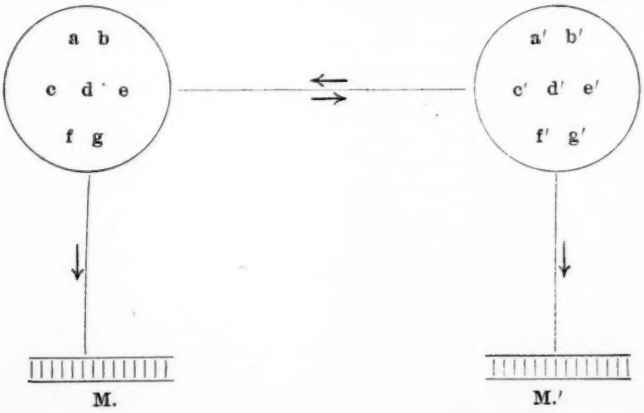
Most of the experiences of the mature man do not have as physiological counterpart the unit of biological activity, the reflex arc, but rather groups of associated processes of the reflex arc type combined in a system of antagonistic forces. The Moral Imperative belongs to such a class of complex experiences. The famous dilemma of Buridan's ass might serve as an illustration, if there were need of one for such a familiar occurrence. Whenever he turned his head towards the water and was about to move, the hay caught his mind's eye and made him look to the bunch, but before decisive steps had been taken, the water was present again. The tilting equilibrium was so perfect in this instance that it never broke.

Moral obligation is never felt but as a member of such an antagonistic system. When an action is performed reflexly or when, being anticipated, it follows smoothly without any conscious inner obstruction or hesitation, without entering into conflict with another end, there is no room for a

[1] "Inductions of Ethics," p. 337.
[2] "Ethics," p. 144.

categorical imperative. If, for instance, I see a person in imminent danger of being run over by a street car and I exclaim suddenly, "Look out!" my behavior does not imply the knowledge of the Moral Imperative. Similarly, if I learn by the morning mail that my dear friend X has fallen grievously ill, and, in a wave of compassionate feeling, I say, "I shall take the first train to him and try to soothe his suffering," the categorical imperative has not been heard. But if, in the first illustration, be it from sheer inertia or from unwillingness to cry out in a public street, I wait a few seconds to see if the person becomes aware of the danger, and then, not automatically, but after a more or less clear knowledge of the unworthiness of my reticence, I direct his attention to the danger; or if, to make the case plainer still, the person happens to be my great enemy and the thought flashes through my mind, "Let him be killed!" while the second after better motives prompt me to interfere, then I may properly be said to have heard the "voice of conscience" in the form of the categorical command. The value of the result of my conduct measured in pleasure or utility is here irrelevant, since we are not discussing the worth of action, but a peculiar consciousness.

The antagonism of these dynamic systems is a *motor* antagonism known in consciousness as an antagonism of intentions, or of ends; *i. e.*, the reflex arc processes are combined in two groups and end respectively in muscles, coördinated for contrary actions; there is consequently a reciprocal defeat of purposes. Schematically the experience may be represented by the accompanying diagram, in which *a*, *b*, *c*, *d*, *e*, etc.,



stand for cortex processes discharging concurrently into the muscles M; and a', b', c', d', e', etc., stand for cortex processes discharging in the antagonistic muscles M'.

As immediately known, experiences of this kind may be compared to that of a man witnessing a kinetoscopic exhibition. On the projection screen he sees a developing tableau; let us say a man at the beach, walking along the spring-board, reaching the end, swinging two or three times and then plunging. Suddenly everything goes out; there is darkness for the space of a second. Then, again, a tableau is thrown on the canvas and is recognized as the continuation of the first; the man is now pulling himself out of the water, etc. The blanks between the tableaux correspond to the apparently empty breaks which we observe between the series of developing apprehensions of an antagonistic ideo-motor system. It feels like a break and a blank, because the apprehensions have a sort of *light* quality, due, no doubt, to their predominantly visual origin; when the cortex processes discharge into motor tracts and the chain of apprehensions is thereby interrupted, the inward panorama darkens and the change in the quality of the experience is naturally felt as a break and an extinguishment: a break, because of the sudden change in the kind of experience, and an extinguishment, because of the absence of representations of visual origin, since motor and visceral sensations now fill the mind. The break between the tableaux is more or less accentuated according as the motor discharge is more or less intense.

So much in the way of a rough psycho-physiological description of the antagonistic reflex arc combinations to which the Moral Imperative belongs.

It is evident that all the experiences of that class do not have the Moral Imperative *quale*; very far from it. Whether I shall, or shall not, go to see my friend may be a matter of simple convenience, there need not be any moral compulsoriness about it. So it is with the protracted dilemma in which Mrs. Smith gets when considering whether she had better put on her magenta or lavender gown. One essential element—the Moral Imperativeness, the sense of obligation—is lacking. Prof. Sidgwick says, in speaking of the subject of the present paper, that he calls such cognitions "dictates or imperatives because in so far as they relate to conduct on which one is deliberating, they are accompanied by a certain impulse to do the act recognized as right." The too broad term "impulse" is not very felicitous in this place, inasmuch as the experience in question is a very specific kind of impulse. Kant aptly described it as a categorical imperative. In order to sepa-

rate that awe-striking experience from others, like it in some respects, we shall have to limit our antagonistic ideo-motor cognitions to such as contain an "Imperative," and not an impulse only. But even this will not be sufficient, because there are cognitions which may well be called imperative and which, nevertheless, are clearly not moral. For instance, the insistent impulse of a sane, or insane, mind to get up at night and see for the second or third time whether the gas is turned out when *feeling quite sure*, as we say, that it is out.

But before endeavoring to differentiate on psycho-physiological grounds the non-moral from the moral-imperatives, it will be useful to single out certain traits of the cognitions having the imperative *quale*, whether it be moral or not.

The illustration just used will serve us well as a concrete case. Suppose, on the one hand, that having gone to bed it occurs to me that the gas in the hall is still burning and that I should go and put it out, but that immediately there comes the rejoinder, "No, it is not burning, I remember very clearly putting it out when I came back from town," the whole matter being thus given its quietus. Or let us imagine, on the other hand, that although I think I have turned it out, the motor-idea to get up for the purpose of extinguishing it recurs again and again, in spite of the momentary assurance I have that it is all right, until weary of the conflict I yield to the persistent "It is burning; go and put it out!" In the former instance, there is no imperative prompting to action, while in the second the impulse is described, in ordinary parlance, as "Imperative." It would be better to use the expression insistent or imperious motor-idea to characterize this class of experience, reserving the term imperative for the *moral* insistent ideas, for the reason that the urgency of the latter differs in a very specific manner from that of the former, as we shall see in the sequel. Nevertheless we shall conform here to a loose usage and avail ourselves of the adjective moral to differentiate the two kinds of insistent motor-ideas. The question before us now is, what is the differentia of an imperative experience?

Two points are in special evidence: the recurrence of the cognition and the definite apprehension of the action to which it prompts. A feeling of unrest and discomfort growing out of the conflict generally accompanies the experience as a derivative. It is worthy of remark that the imperative process need not be characterized by great motor intensity, although it always has a clear efferent conclusion. Oftentimes the calm, ineffective way—ineffective, because it does not have in itself the power to carry out its orders—in which it makes itself known, seems to be out of all proportion with the motor

energy with which it is resisted, and yet its teasing persistency may compel obedience. This point will assume a considerable importance when we come to consider the Moral Imperative.

The reader may have noticed that thus far we have had nothing to say regarding the will. As our problem is neither action, nor the relation of the ego to the Moral Imperative, but the psycho-physiology of the Moral Imperative itself, we shall not have to take the will into consideration at all. Whether we deal with a simple impulse, or with an insistent idea, or with a Moral Imperative, the will is excluded, for all these experiences are involuntary activities of which we find ourselves possessed and upon which we may react, but which we do not will into existence.

And now let us address ourselves in earnest to the consideration of the problem already mentioned and for which we have prepared the way in the preceding pages. In what do the consciousness and the physiological mechanism of the moral law differ from that of a "physical" or "external" compulsion, or from that of an imperious idea?

Ready common sense is not long held in suspense by such a query; it answers, "it is the object of duty, the things that are right, that we feel as morally binding, just as red things are seen red." Unfortunately for our peace of mind we do not believe that this *petitio principii* is the final word of science; we want to know what it is that makes a course of conduct to be felt as a duty, as morally binding. The feeling in itself is evidently an ultimate datum, but we may point out its qualitative relations to other feelings, and we may legitimately seek to do that which has already been done in the psycho-physics of the simpler sensations, of tone for instance, by Helmholtz, and even in the case of such complex experiences as the emotions,[1] *i. e.*, we may seek for the specific mechanism which is the necessary counterpart of the "Stern Daughter of the Voice of God." To do this is properly the business of psycho-physiology.

Our end will be more safely reached through a critical examination and comparison of typical cases of moral and non-moral imperative experiences. We shall choose illustrations as similar as possible.

I. Being in bed, I am suddenly made aware of an impulse to go and put my watch, which against my habit I have left in my waistcoat pocket, in its accustomed place under my pillow. As soon as I have realized the meaning of the

[1] See Darwin, Wm. James, Lange, Dumas, etc.

prompting, a counter impulse sets in, vaguely preceded by the apprehension of the comfort of the bed and discomfort of leaving it. For an instant I remain relatively quiescent; meanwhile, the apprehension "watch-in-my-pocket-not-under-pillow" develops; I see the usefulness or the uselessness of its being under my pillow and the danger of its remaining in my pocket, etc. Suddenly a break happens in the mental scenery and I say to myself, "The watch is safe where it is; I do not need it under my pillow; it is just as well where it is." But this has hardly crossed my mind before the first impulse is present again. For the second time, as soon as I have realized its meaning, I veto the getting-up tendency, judging that there is no need of my leaving my comfortable bed for such a purpose. But no amount of considering and reasoning prevents the impulse from recurring again and again, until exasperated I jump up and execute the order.

In this experience, we must notice three "movements" of the reflex arc type: (1) the impulse to get up following immediately, quite involuntarily, upon the apprehension of watch-in-my-pocket-not-under-pillow. Whenever that thought pops up, the impulse to get up follows mechanically as its efferent conclusion. The action is known through the impulse and not before it. (2) As soon as the meaning of the felt impulse is realized, an antagonistic movement occurs, the afferent part of which is more or less dimly known as an apprehension of discomfort, of cold, etc., that would be experienced were we to get up, and the efferent side of which is the innervation of muscles antagonistic to those stimulated in 1. This second reflex arc is, like the first, involuntary. (3) When impulse 1 has been neutralized by innervation 2, the necessity, usefulness, consequence, etc., of 1 and 2 cross my mind and, in a motor tendency antagonistic to getting up, I say, "I need not get up; it is a matter of indifference whether the watch is here or there." This third movement differs from processes 1 and 2 in that it is *reflective, i. e.*, while in movements 1 and 2 the action was not known before the impulse to perform it, here the apprehension itself gets its cue from, or starts with, the cognition of the deed; the action is, as we say, *under consideration*. In the present instance, the motor conclusion of the third process is antagonistic to getting up, the imperative impulse is not approved of.

A group of three reflex arc processes—the first two involuntary, reflex, ending in antagonistic motor tendencies; the third one reflective, ending in the disapproval of the insistent impulse—linked together by associative connections and repeated an indefinite number of times, makes up the

experience we are now analyzing. To say that this triune process remains the same throughout its repetitions does not mean that the same apprehensions are repeated, but rather that the motor results are similar and that although the apprehensions may differ somewhat in vividness and in contents, they are, nevertheless, recognized as referring to the same objects and to the same action.

II. In the middle of the night I am awakened by the coughing of my brother lying sick in the next room. My first movement is to go to him and see whether I may do something for his comfort. Before this action is carried out an antagonistic impulse arises, following upon a more or less obscure apprehension of bed-comforts, etc. But this shrinking from getting out of bed has hardly subsided before a chain of considerations passes before my mind: the pain my brother might be enduring, which I might relieve; the meanness of my aversion to move, etc.,[1] a chain of apprehensions ending in a motor tendency in harmony with the first, and which I may express to myself in the words, "Yes, you *ought* to get up; it is your duty, it is right that you should do so." But somehow or other I remain inert until another fit, or the memory of the first fit, of coughing, or the thought of my brother, starts up again an abortive attempt to get up, etc.

Here, again, we have three coördinated arcs which may go on repeating themselves indefinitely. The first two are in every respect similar to the corresponding processes of the preceding instance. They are both involuntary, impulsive. The third movement, like the corresponding one of instance No. I, is reflective, but it differs from it in its direction; instead of opposing the motor conclusion of the insistent idea (the first reflex arc), it coincides with it.

III. Let us suppose that instead of being sick, my brother is in health, and that violent coughing from him still determined in me an impulse to go to him. The deterrent reflex arc process takes place as in illustration 2. Instead of imperatively urging me to attend upon my brother, the third process now leaves me free to do what I may choose; I express myself mentally in the words, "I need not get up;

[1] Before going further we desire to draw the attention of the reader to the fact recently pointed out by Prof. Dewey in his article on "The Reflex Arc Concept," *Psychological Review*, Vol. III, No. 4. That which is usually taken as a reflex arc is generally a coördination, or combination, of a number of reflex arcs. The apprehension side of our reflective processes is in fact composed of a lot of afferent-efferent processes; but they are so combined that we may consider them as one resultant reflex arc movement.

there is no urgency; he does not need me; I may remain in bed if I like it best," etc.

IV. Fourthly, let us imagine, that, again in bed, I suddenly remember having neglected to write a letter recommending some one for a position. My first movement is to get up. Then comes an antagonistic innervation following upon the apprehension of the discomfort involved in getting out of bed to sit at the writing-desk, etc. Presently I become clearly aware that I have promised to write the letter, that the young man trusts in my word, and many unnamable, ominous shadows hover past my field of mental vision, and I say to myself, "Get up; you ought to." This experience is in every essential point similar to II.

The reader has no doubt noticed that the reflective motor-apprehension of II and IV was categorical, no alternate was left, "You ought to get up" was the conclusion; while that of I and III is best described as disjunctive with regard to the action suggested by the first impulse: "There is no need of your placing that watch under your pillow," I said to myself; and, if I had fully expressed my attitude at that instant, I should have added, "There is also no necessity for leaving that watch where it is, it is a matter of indifference," *i. e.*, the experience II (the sick brother), and IV (the promised letter), include the moral categorical imperative, the voice of duty; while I (the watch), and III (the brother in health), are morally indifferent. All four experiences are similarly composed of groups of three coördinated processes of the reflex arc type; two of them being reflex and the third reflective. If our analysis is right, it must be in the reflective process that we must look for the "moral" differentia, inasmuch as a reflex movement cannot be fraught with the feeling of moral oughtness. The words, "You ought or ought not to do this," and, "Do what you please about it," designate like every other word or group of words, by means of sensory signs, certain experiences, sensations, presentations, representations, apprehensions, feelings, emotions, etc., *i. e.*, certain irreducible, immediate consciousnesses. In the instances in question, these expressions are the names given to the motor conclusions of the reflective apprehensions. When those specific processes are felt in the described relations, we say either that we feel the oughtness, the duty; or that we feel the action to be in itself indifferent. What the difference thus expressed is can be brought out only by a careful examination of the reflective motor-apprehensions themselves. As the process is of the reflex arc type, we shall consider it under two heads: the afferent and the efferent side.

As to its afferent side it is similar in the moral and in the non-moral movements; in both cases, it is a reflective as opposed to a reflex apprehension, *i. e.*, it takes its cue from the cognition of the action of which we have become aware through the reflex movements 1 and 2. Thus we know, as we say, that we are considering the advisability of a certain action, hence the name *reflective*. The efferent conclusions of these reflective arcs end in the muscles that would be needed to carry out, or resist, the execution of the action, or in their speech substitutes. We may call the reflex movements *blind impulses* because consciousness lights up only the way that has been already traveled; and bestow upon the reflective processes the name *intelligent impulses* because the lamp is turned forward and lights up the end at stake.

We have used the term *reflective* and not *voluntary*, for the evident reason that a voluntary activity does not only include the prevision of the possible motor conclusion, but also a something else, for which we have no better name than the ill-understood word "effort." The motor conclusive of the reflective process with which we are dealing comes not unannounced, but unasked for; be it desired, willed against, or unwilled, it imposes itself upon us just as much as the reflex process. This unwilled experience may very well be followed by a voluntary activity striving either for the suppression or the fulfilment of one or the other of the recurring movements, or yet by deliberate efforts to consider the question thoroughly from all sides; but this is not at all necessary to a reflective process of any kind, and especially not to a moral-imperative experience.

A second and far-reaching characteristic of the afferent side of the reflective arc, belonging also to both moral and non-moral experiences, is its independence of passions, emotions and sentiments; or to express this physiologically, its independence of the sympathetic nervous system. It appears to be an activity limited to the cerebro-spinal system. For the sake of convenience we shall anticipate a little, and, before having brought in all the evidence, shall make the statement that whenever the Moral Imperative is felt, it is the correlate of a purely cerebro-spinal reflective motor-process. Let the reader accept the statement on probation and keep it in mind through the rest of the paper; we trust that he will be finally convinced of its truth. We say that the third movements in our illustrations develop passively, dispassionately, coldly. Emotion and strong feeling may well develop after the motor-conclusion has been reached; it is this occurrence which often veils the fact we are trying to

bring out. Let the reader recall, for instance, the shame which will follow the realization of the command, "You ought to get up," when alongside of it creeps the consciousness, "You are too mean, or too lazy to obey," or when the command is looked upon as from God, and in consequence a mass of ethico-religious feelings is stirred. The exclusive cerebro-spinal dependence of the Moral Imperative has been also obscured because other experiences dependent in part upon the sympathetic nervous system have vaguely been identified with it under the general name of "moral-feeling." If, for instance, I recoil in shame and disgust from a lascivious piece of art, just as I push away a sour beverage, the turning away from and the being ashamed and disgusted do not imply the experience of the Moral Imperative, although a tendency to turning away might be its conclusion.

We pass now to the consideration of the efferent part of the reflective movement. Thus far we have found nothing differentiating the non-moral from the moral reflective process. In illustrations II and IV, the reflective motor conclusion was expressed by the words, "You ought to get up, there is no other alternative." In I and III, on the contrary, the motor side was not felt as a necessity of performing a particular act. I felt, "You need not get up unless you choose; do what you please." These expressions indicate, it seems, that, whereas concerning the first, the cortex activities corresponding to the reflective apprehensions pool their forces and discharge harmoniously in coördinated motor channels, concerning the second, the cortex stimulus either loses itself in many non-coördinated motor tracts or divides itself along two antagonistic lines of discharge, with the result that in the former case there is a clear, definite apprehension of the action, together with an impulse; while in the latter, the representation of the action is usually less vivid and there is no discernible tendency either to perform or resist it. The very definite aversion to getting up felt, for instance in illustration I, must not be mistaken for the motor side of the reflective arc process. A careful introspection will reveal that it is rather due to a repetition of the second reflex arc, occurring so quickly after the reflective arc that it may be thought to belong to it.

From the foregoing analyses, we conclude that experiences II and IV, *i. e.*, those including moral obligation, differ from the non-moral ones, I and III, in that the third, or reflective process, is imperative, which means in physiological terms that its efferent discharge is definitely organized along coördinated motor tracts. The "Moral" arc in these cases is: (1) reflective; (2) wholly cerebro-spinal; (3) it has a clean-

cut, coördinated motor conclusion prompting to an action. While the corresponding reflective arc of the non-moral experiences, if identical in regard to points 1 and 2, differs in the manner above stated with respect of its efferent termination.

The illustrations from which we draw the foregoing conclusions were expressly chosen in order to differentiate the Moral from the non-Moral Imperatives; we shall therefore have to extend the range of analysis before we may legitimately generalize our findings and affirm that these three characteristics belong exclusively, and always, to the Moral Imperative experience. Let us consider rapidly other typical experiences and see whether they do or do not fall in line with the preceding ones.

a. The thought of my friend X flits through my mind and I am prompted to go and see him. Then comes a string of apprehensions developing around "my friend:" some pleasant moment I spent with him not long ago, the attractive look of his study, his good cigars, his cordial greeting, and the warm pleasure he took in the conversation. Here a little wave of pleasurable feeling or emotion interrupts the chain of apprehensions and I feel an impulse to go; "I shall go and see him," I say. Presently another series of tableaux passes before my mind: he gave me three months ago an essay of his which I have not yet read; at our last meeting he inquired what I thought of it. He looked a bit ruffled that I had so long postponed the reading of it. I see him now betraying his displeasure at hearing that I have not yet found time to read the paper. Another wave moves me and I recoil, thinking, "I shall not go."

We may imagine the process coming to an end after the first or after the second ideo-motor movement; or yet continuing after the third along similar lines of associations, and ending now in a "going" and now in a "not-going" motor feeling. The first afferent-efferent process is automatic; the second and third are reflective, but neither one is free from the influence of the organs of the vegetative life; the chain of apprehensions is cut short by an influx of stimulus from the sympathetic nervous system. This experience does not contain the Moral Imperative.

Let now the circumstances be changed; suppose that X is of a humble station in life. He has called upon me long ago and I have not yet returned his call; he must feel slighted, etc. The motor conclusion comes and is expressed by the words I pronounce mentally, "That is wrong; you ought to go; it is your duty." It might be that the thought of having failed in civility from such a mean motive towards

a man I esteem shames me, and that I decide to go on the spur of this emotion. If such is the case, I do not feel under moral obligation; for acting simply because we should be ashamed to have people know that we had not acted, does not, according to general experience, fall within the field of the morally obligatory. But, and this must neither be overlooked nor misinterpreted, I may feel both the shame and the moral obligation. As a matter of fact shame often follows the feeling of oughtness, but it never precedes it as its *determinant*. Even when emotions or feelings precede or follow immediately the "voice of conscience," we are fully aware that it would be heard just as imperatively without them; this is due to the fact just stated that emotions form no part of the "*moral*" arc. In an experience like this, after the first realization of "oughtness" and of shame, these two psychic states generally recur both in representation and in presentation. In representation they "feel" at times as if they penetrated each other and made one more or less homogeneous state. This apparent fusion easily confuses the experiencer in the belief that "oughtness" and shame are here inseparable from each other.

In this last case, the process is reflective, free from feelings and emotions, and its efferent conclusion brings before the mind definitely and urgently an action. Just as in the previously considered cases, this ideo-motor Moral Imperative does not stand alone, it is correlated with other ideo-motor processes.

b. We hardly need dissect Moral Imperative experiences starting with the sting of the grosser passions, hunger, lust, etc. They would be found to be analogous to the preceding cases. We should find first a reflex impulse tending to the gratification of the passion; then, possibly, a chain of apprehensions including sentiments and emotions, such as fear of discovery, shame at our weakness, etc., ending in an assent to, or a dissent from, the deed. If moral obligation is felt at all, it would be found again to be the return feeling of the efferent conclusion of a reflective non-vaso-motor arc.

c. I have been publicly criticised in a spirit of jealousy by some one, who, to reach his end, has not shrunk from false and calumnious representations. In my anger and indignation, I resolved to answer in a withering letter. I may feel violently impelled to write, without at all being conscious of moral obligation. If I cool down for a length of time sufficient to allow the undisturbed development of the cortex processes—and that is a very short time indeed,—it might be that the motor conclusion of the corresponding chain of apprehensions points nowhere, leaves me indifferent. If it be

so, I shall not know the Moral Imperative. But if it ends roundly in coördinated motor channels, filling my mind with the cognition of one specific action, then I shall feel moral obligation.

d. There are cases in which moral obligation is felt, but is not accompanied by the apprehension of a specific action. A general on the field of battle, for instance, may feel it morally imperative to make some move, although no particular movement seems best. Here the apprehension of the certain destruction which, under any conceivable possibilities, will overtake the army in its present position has a clear, powerful, motor conclusion tending to action again, but not to any highly specialized action. It is a discharge along the motor tracts coördinated for general bodily activity, turning about, running away, etc., or for the substitutes of these motions. Here again the spirit of the description we have given of the moral imperative process is not at fault; the motor discharge is not a random one, it does not reach as well the flexors as the extensors; it is coördinated for general action. The commander may subsequently consider a number of possible movements, find them all destructive and recoil from them all, until perplexed and confounded, he may grow cold and stupid,—but this has naught to do with the Moral Imperative.

e. The cases of so-called *external compulsion*, be it psychical or physical, do not differ from the preceding ones as far as the psycho-physiology of moral obligation is concerned. We need not consider the instances in which the habit of implicit obedience is so deeply ingrained that any order whatsoever is immediately executed or accepted without first awakening opposition or reflection. Such an automatic obedience as that of the old soldier at the Invalides who dropped his dinner which he was carrying in a tray on hearing a practical joker order "Shoulder arms!" evidently falls outside of our study. There is another sort of external compulsion affecting those who have learned the uselessness of resistance to certain commands coming from certain persons, or to certain despotic passions, or to the force of circumstances. Here the awareness of the compulsion does not always prevent the anticipation of the consequences of complying, and at times there is a recoil from them. When this recoil proceeds from a desire to avoid the pain or displeasure that obedience would occasion, the moral command is not heard. If we are aware of the Moral Imperative at any stage of an experience of this kind, whether we try to resist, or give up in despair without attempting resistance, we shall find on investigation that it is the conclusion of a reflective cerebro-spinal motor process, exactly as in the preceding cases.

f. The "logical" activity through which we arrive at scientific knowledge presents many of the characteristics belonging to the Moral Imperative. It is a reflective activity independent of the sympathetic nervous system and its conclusion has a finality, a "categoricalness," akin to that of moral obligation. Galileo would have been glad to persuade himself that the earth is motionless, but the logical imperative protested and affirmed, "yet, it *does* revolve around the sun!" Despite these similarities, people find no difficulty in practically differentiating between these two classes of experiences. The conclusion of the Moral Imperative process urges to a specific action, affecting some being, while the other pronounces upon the existence, past, present, or future, of an object of thought. Physiologically this means that in the logical process the efferent side of the reflective arc does not discharge in muscles coördinated for an action *that would modify the experiencer's relation to some existence*, although, to be sure, it has also a definite motor conclusion.

We have said of the reflective non-sympathetic process that it is impulsive, that it prompts to action. But its impulsiveness is peculiar; it differs from that of the reflex and of the sympathetic experience in that it does not seem to be due to the putting in incipient activity of the muscles involved in carrying out the action. Close introspection reveals, it seems to us, that the moral imperative process ends efferently, not in the muscles of the action itself, but in those of the speech organs which stand as their representatives. It happens thus that the urgency is not to perform the deed, but rather *to state that fact in speech signs*: we are not incipiently performing the deed, but, instead, we utter or hear mentally, "You ought to do this," or words to that effect. We shall postpone to another part of our paper a fuller discussion of this very interesting point. Let us add only that this impulsiveness is generally followed by an innervation of the represented muscles themselves and, on that account, we usually have, coming after and not belonging to the knowledge of the Moral Imperative, a genuine impulse to perform the action itself.

We may now go back to the provisory differentiation on page 539 and, on the strength of the agreement of cases *a*, *b*, *c*, *d*, *e* and *f*, with I, II, III and IV, we shall set down our thesis in the following general terms, referring the reader to the sequel for additional confirmation:—

The Moral Imperativeis the psychic correlate of a reflective, cerebro-spinal, ideo-motor process, the efferent part of which is organized into motor tracts coördinated for a specific action. No Moral Imperative can be known in the absence of such a

physiological activity; and whenever an activity of this kind is known in consciousness, it is known as a Moral Imperative. This reflective arc stands in dynamic inter-relation with one or more arcs that are either non-reflective, cerebro-spinal, or sympathetic reflective, or non-reflective sympathetic. These associated processes vary widely in number, vividness and compulsoriness; the only necessary thing is that there be at least one such process preceding the "Moral" arc and forming with it a dynamic system.

The four diagrams on the opposite page may help to give definiteness to our thesis.

Explanation of Diagrams. In every diagram the dotted line *C.* stands for the cortex layers of the brain, and the dotted line *Md.* for the midbrain and the medulla. *M.* stands for the voluntary muscle system, exclusive of the speech organs; *V.* for the organs of vegetative life generally; *a. b. c.* for those parts of the brain concerned in the apprehension of the action, and 1, 2, 3, 4, 5, 6, for the apprehensions developing from, or ending in that of the action; *vs. c.* for visceral sensation centres; *s. c., s'. c'.* for speech centres, and *s. o.* for speech organs.

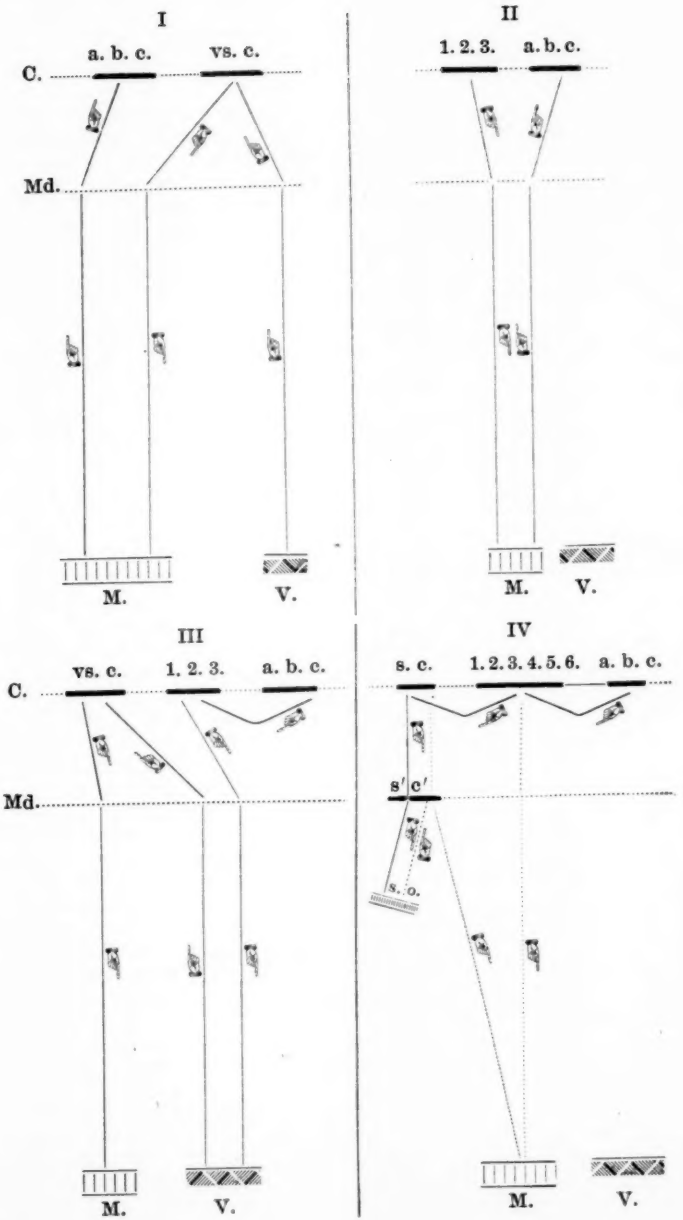
I. *A sympathetic reflex process*, starting in *V.* The action, to which the efferent discharge to *M.* prompts, is known only after the return motor sensations.

II. *A cerebro-spinal reflex process.* It may stand for the non-moral imperative processes we have instanced. It starts with 1, 2, 3, 4, the processes corresponding, for instance, to the apprehension of "watch-in-pocket-not-under-pillow," and ends with *a. b. c.*, which stand for knowledge of action towards which an impulse has just been felt.

III. *A sympathetic reflective process.* Sequence of events: (1) *a. b. c.*, (2) followed by a series of associated apprehensions. 1. 2. 3. (3) Discharge along visceral motor tracts, into viscera *V.* (4) Return visceral sensations in *V. S.* (5) Motor discharge in cerebro-spinal system reflexedly felt as impulse to act.

IV. *A reflective cerebro-spinal process.* Sequence of events: (1) *a. b. c.* (2) 1. 2. 3. 4. 5. 6. (3) Discharge into the so-called speech centres *S. C. and S'. C'.* (4) Sensory return.

Remarks on Diagrams III and IV. The class of experiences represented by schema No. III is the one with which we are most familiar; it is the "natural" experience *par excellence;* the only kind of experience possible until the sympathetic nervous system has been differentiated to a high degree from the cerebro-spinal system. We must take especial notice in this class of experiences of the influx of stim-


I
a. b. c.
vs. c.
C.
Md.
M.
V.
II
1. 2. 3.
a. b. c.
M.
V.
III
vs. c.
1. 2. 3.
a. b. c.
C.
Md.
M.
V.
IV
s. c.
1. 2. 3. 4. 5. 6.
a. b. c.
s′ c′
s. o.
M.
V.

ulus due to visceral activity; it precedes the cerebro-spinal motor discharge which brings about the action, or the impulse to act, and constitutes a weighty factor in its determination. The intellectual consequence of this nervous influx through the sympathetic system is to shorten the string of reflective apprehensions, *i. e.*, to cut short the development of what may roughly be called the "reasons" for acting. There are occasions when the experiencer is gallingly aware of this arrest; for instance, when, in spite of vigorous dislike, we strive to be impartial towards some one, and again and again discover that the affecto-emotional wave has led to a judgment before we had taken into account this good point and that estimable trait. It even happens that under the influence of disdain, anger, impatience, pride, self-conceit, etc., we reach a conclusion-motor-impulse, all the while (as it seems) knowing that we have suppressed certain evidence, and distantly protesting against this unfairness. The blinding effect of passion has long ago, and in many lands, been turned into popular sayings. How this must be, and within what limits, can easily be understood from the point of view of dynamic physiology. How this state of things affects our general estimate of the worth and dignity of the judgments reached through processes of this class, we shall see later on.

It hardly need be said that the class of experiences represented by schema No. IV can belong only to beings whose cerebro-spinal nervous system has been so far differentiated from the nervous system of the life of nutrition and reproduction that it has isolated, closed, afferent-efferent channels. This schema represents all the higher purely "intellectual" operations; the calm deliberations of a man of science as well as the "voice of conscience." How the former differ from the latter we have attempted to say on p. 543, case *f*.

For the sake of brevity and simplicity we have spoken of 1, 2, 3, 4, 5, 6, as if they represented direct, successive associations of purely intellectual representations. We incline rather to the belief that every one of the links of the chain of apprehensions is made up of a "shortened" reflex-arc process,[1] and that it has frequently a well-marked affecto-emotional tone. But these reflex-arc processes get into dynamic unity and pool their efferent sides into coördinated channels, and so we may deal with them as if they made up one single reflex-arc.

Our use of the expression "purely intellectual" will probably have suggested an absolute independence of intel-

[1] See the paper of Prof. J. Dewey, already mentioned.

lectual knowledge from pain, pleasure and emotions. The observations made with the help of the sphygmograph, plethysmograph, pneumograph, dynamometer, and otherwise, have placed beyond doubt that which the anatomy of the nervous system had already suggested as highly probable, *viz.*, that even the mental activities which feel most purely intellectual, exercise a measurable influence on blood circulation and, generally, on visceral activity. But we may, legitimately, it seems, make use of the expression "purely intellectual" to name those cognitions in which the experiencer does not recognize an affective tone, or an emotion. The faint and unfelt, or rather undistinguished, general bodily echo of the felt intellectual process does not concern us here. Moreover, as introspective observation declares that the Moral Imperative is the purer, the more exclusively intellectual it feels, we need not in a study of this phenomenon take into account an element that is to it as the dross to the metal.

Another explanatory remark we must make concerning the efferent conclusion of this IV type of process. It is marked on the schema as ending in the speech organs, and not in *M.* as in the others. We have elsewhere had occasion to bring out a rather subtle difference in the "feel" of the Moral compared with the non-moral imperative process. We found that the being aware of the moral imperativeness of an action is not, strange as it may seem to the superficial observer, synonymous with the consciousness of a motor impulse to perform it; that, unfortunately for many of us, the knowledge of duty is not necessarily accompanied with an impulse to act. We are frequently, to use the words of the theologians of the 17th century, famous through the Letters of Pascal, in possession of the *grâce suffisante*, but we have not the *grâce efficace*. It lays down an absolute law, but it does not carry it into execution; it is a purely legislative and not an executive power. Hence it is that its imperativeness does not infringe upon our "freedom." This is to be physiologically interpreted, we believe, as meaning that whereas in the non-moral imperative processes the efferent discharge reaches the muscles that would carry out the deed itself, in the case of a Moral Imperative it is vicariously directed to the speech centres and thence to the speech organs, which act as substitutes, and therefore we have an impulse to *speak*, but not to do the deed. This substitution will appear more plausible, and its *modus operandi* more intelligible, if we glance at the psycho-physiological history of "consent," or "approbation."

Originally consenting to something is *doing* the thing consented to. The amœba either encloses and absorbs the parti-

cle that touches its body, or does not; the newborn infant sucks, or does not suck. But very soon substitute motions are developed, and take the place of the action itself: the turning away of the head gradually takes the place of the tight shutting of the mouth. Later, when speech makes its appearance,a sub-substitution occurs; the child instead of closing its mouth, or turning away its head, speaks the vicarious "no," and, henceforth, for the deed itself is substituted a representing deed of the speech centres and organs. To follow, with much detail, the reflex-arc mechanisms through their exceedingly complex schemes of substitutions would be a long and difficult task; what precedes may, perhaps, be adequate to our purpose. It will, at least, suffice for a correct understanding of the last clause of our definition of the Moral Imperative process.

Many will find it hard to admit that the less the Moral Imperative experience contains an impulse towards the execution of the command, the clearer it is. This difficulty is caused, we believe, by the fact that the substituted activity in *s. c., s'. c'. and s. o.*, is probably always followed, according to the laws of association, by a motor discharge, more or less vigorous, into the represented muscles; this impulse is wrongly taken to be an intrinsic part, or a condition, of the consciousness of the Moral Imperative, although it is only an appendage.

The dotted line going from 1, 2, 3, 4, 5, 6 to *M.*, in Diagram IV, represents the possibly generally present survival of the primitive motor discharge going directly from the sensory realms to the deed-muscles themselves. We need not expect that the primitive sensory-motor course is absolutely obliterated in moral-imperative processes; they need not be pure.

We have all along proceeded on the assumption that the cognition of moral oughtness is the psychic side of certain particular processes of the reflex-arc type. We have said that the words "you ought to get up" or "do what you please, it is indifferent," are the names of the consciousness of such physiological activities. We shall probably be called to account by some for setting aside so unceremoniously the theory that used to reign supreme among ethical philosophers, and which is still in high honor in many quarters, not so much because of the satisfaction it gives as because no other theory, more in harmony with the conceptions of modern psychology, has been clearly set forth. The knowledge of duty, of oughtness, they will say, precedes and is the determining cause of the bodily activities which you put forward as the condition of the cognition of the Imperative.

This theory is an old acquaintance; it is the child of the now obsolete conception of the soul as an entity knowing, somehow, independently of bodily activity. Little by little it has had to give up some portion of the field of experience; at first the lower cognitions, those of the senses; more recently, the emotions were taken out of its realm by the Lange-James theory. In looking upon the moral feelings, as conditioned by specific physiological processes, we do no more than place this class of phenomena under the conception now generally accepted touching the relation of mind and body. We do not say that these bodily activities *cause* the "oughtness," neither do we say the reverse; we are content to state a parallelism and to affirm that, without these physiological phenomena, there is no possible knowledge of the Moral Imperative, just as—and for similar reasons — there can be no sight without the healthy activity of the optic nerve and of certain definite parts of the brain. Arguing on this point would be to enter upon a discussion of the most fundamental conception of modern psychology. To do this would be an impertinence unless we could show, or had sufficient reason to believe, that the facts of the moral life cannot, with reference to this conception, be classed together with the lower sensations, with the emotions and with the æsthetic feelings. But, of this, not only is there no evidence, but, moreover, all the facts known make it absolutely impossible to sever the moral from the rest of the psychic life.[1]

In so far as we call actions *right* or *wrong* according as they do, or do not, agree with the leadings of the Moral Imperative, and as we use the word *duty* to designate the actions morally binding upon us, in so far do these names *right*, *wrong*, *duty*, derive their meaning from the consciousness of the Moral Imperative, and are they applied solely under its guidance. We say "in so far" because we are not now prepared to affirm that right and wrong, in their moral meaning, as understood in civilized countries, never have another origin than the experience of the Moral Imperative.

The proposition that the knowledge of "oughtness" or of "duty" is the result of the described reflective arc is appar-

[1] For the rest let it be said that the parallelism doctrine is to our mind thoroughly unsatisfactory: it supposes a chasm between events we feel the need of relating in a sequence. The postulation of a causal relation between the two classes of events, in either direction, will remain unintelligible as long as our understanding of the nature of these two series of facts does not change. Let us hope that psycho-physiology will soon stand on a surer foundation.

ently controverted by innumerable experiences. The popular mind will be quick to poin tout the undeniable fact that we frequently know things to be objects of duty before having at all considered their consequences, purport, etc. (1, 2, 3, 4, 5, 6, in Diagram IV). If I think of stealing, I immediately know that it is my duty not to steal; and to say that this cognition follows upon a long series of apprehensions as before described, is most clearly a misrepresentation of the sequence of events. I know first that it is duty, and if I have the thoughts in question at all, it is only subsequently.

Psychology finds the answer to this criticism easy. Sensations are *signs* of objects, said Helmholtz. A sensation may stand for other sensations, and a feeling for other feelings. The wealth and complexity of our psychic life are largely due to the possibility of our knowing, on experiencing a sensation, or a feeling—if we may speak thus—that its object would under different circumstances give us other sensations and other feelings. Because of such substitutions we are enabled, on the perception of one quality, to apply to its object adjectives which were given to it to describe some other qualities. For instance, if on seeing an apple I know it to have an acid taste, a certain color has given me the knowledge of a taste. Similarly, because of the name of a famous painter on a canvas, or because a connoisseur declares it a work of high artistic merit, I "know" it is a beautiful painting, I believe it, I say so to my friends. I have neither tasted the acidity of the fruit, nor felt the beauty-thrill of the painting, and yet I know the one to be acid, and the other to be beautiful. It is evident that although I know, now, acidity and beauty without really experiencing them, it is only because some time ago, in my early experiences, I have tasted "acidity" and seen "beauty," and associated these complex verbal signs with other sensations, feelings or perceptions, due to the same object. Thanks to this association I am enabled, in subsequent experiences, to conclude immediately from the color to the taste name, without going through the long process of biting the apple and the physiological activity that follows. It is a similar vicarious substitution that makes it possible for me to-day, on the recognition of certain non-moral traits, to know that an action belongs to the class "*Moral Imperative.*"

We must not overlook the fact that not only do certain simpler, more convenient experiences substitute themselves for others in the same individual, but that, moreover, in the case of gregarious individuals, the experience of one of them often introduces among the others the name of that experience and certain forms of activities which were, in the orig-

inal experiencer, the reaction to it, although they may never have had the experience they name and under the influence of which they now act. Among the many who speak of the "awfulness of sin" and who live in fear of it, a comparatively small number have really experienced the "awfulness of sin." Shall we instance belief, faith, or the many virtues which we practice as the result of imitation? In its lowest, as in its highest regions, this vicarious world of ours is indebted to a few geniuses, blessed with exquisite sensitiveness and intelligence, and favored by circumstances, for the possession of substitutes—in the form of names or other sensations and feelings—for certain experiences that have never been their own, and for the habit of properly reacting to them. This is practically all that is necessary for the conservation of the individual and the progress of the race. These geniuses have bequeathed to the common herd their æsthetic and moral judgments and habits.

Frequently in the course of education, after having first learned the substitute and the reaction it calls for, we become acquainted with the substituted. This practically invaluable substitution, by which the original sequence of events is changed and links are dropped, is the great mystifier of those who attempt to unravel the mysteries of psychic life. As long as the representing is still able to call up the represented, the difficulty thrown in the way of the investigator is comparatively small; but when through long existence of the shortened connection and long disuse of the represented, the links are lost, then Nature, which seems to take pleasure in making safe her secrets, triumphs, and man struggles long in vain with problems many of the original features of which have completely faded out.

Let us now, changing our line of argumentation, search more closely for the psychic characteristics of the Moral Imperative as we find it in ourselves and as it has been described by those who have dealt with it in writing, and then ask ourselves what sort of physiological process might condition that experience. We shall thus discover the possible justifications of the current erroneous theories of Moral obligation, and show that, were we conscious of a reflective motor process of the kind described on page 543, it would necessarily be—according to our present knowledge of psycho-physiology—through the very qualities that we shall put down as characteristic of the Moral Imperative.

The poets speak of the Moral Imperative as the "Stern Daughter of the Voice of God" (Wordsworth). The ethical and religious philosophers define conscience 'the voice of a

conscious being external to me, to all men, who has made us and all the world' (Martineau); and the guiding genius of the metaphysical philosophy of this century cautions us against "allowing ourselves to think of deducing the reality of this principle from the particular attributes of human nature" (Kant). The Moral-Imperative, according to him, is to be clearly differentiated from "propensities," "inclinations," and everything else belonging to the individual. It transcends experience, it is a law, universal, categorical and imperative.

They all, poets and philosophers, struggle to express in the loftiest and the most uncompromising terms the universality, impersonality, finality, obligatoriness and imperativeness of this experience. After its imperativeness, it is its objectiveness, its independence of throbbing flesh which has attracted the special attention of the moralists. They have found a wealth of expression to picture that quality of the low but stern voice heard in the quiescence of the soul, in the stillness of the night; always there when we listen for it, but never heard in the din of passion. When the fever of jealousy, anger, or lust, surges on, or when vanity, distress or anguish thrills our souls, we feel the imperiousness of non-moral imperatives—of desires and cravings; but it is only when the wave has passed, when calm is reëstablished, be it only for a second, that we hear the proclamation of the moral law in that peculiar, impersonal, unemotional, toneless but autocratic voice that is taken for the voice of God.

Another quality of the Moral Imperative is best felt when it stands in antagonistic combination with processes containing elements from the sympathetic nervous system. It seems, then, to occupy the equilibrated, the stable, the rest position. To pass from it to the other processes feels like being jerked away from the resting attitude; and to come back to it seems like returning from an unstable to a stable position. To this peculiarity, we must add the fact that we tend to return to this calm, passionless state. It is, among the ideo-motor processes, as the tonic note of a melody; we are really satisfied only when it is the last "feel" of the complex experience. It would be more exact to say that when we return to equilibrium and stability—as we necessarily must—then is the time when the Moral Imperative looms up.

Finally, let us add to these traits the peculiar inefficacy which we have mentioned and accounted for as best we could, and we shall have a sufficiently complete description of the categorical imperative: *impersonality*, *universality*,

imperativeness, obligatoriness, passionlessness, finality, stability, inefficacy, are its essential characteristics.

Does the particular physiological process we have described as the condition of the Moral Imperative account satisfactorily for these qualities, on the basis of analogy and of accepted psycho-physiological theories?

Concerning the stability and the fact that the Moral Imperative recurs together with the consciousness of rest and equilibrium, it has no doubt occurred to the reader that it is a consequence of the absence of stimulus from the sympathetic nervous system. We can feel at rest, stable, only in the absence of emotions and feelings, because the affecto-emotional states in general are, in their very nature, movements. Cerebro-spinal processes, it is true, are in some respects just as much transient as affecto-emotional states, but their coming and going seems to be as in an horizontal plane, while the movements of affective cognitions are as in a vertical plane, *i. e.*, while in the latter case there are necessarily gradual changes in bodily tensions, in the former such tensions are either not present or unessential. It is not easy to find adequate words to describe such a quality as this; but every one able to turn his vision inward will readily apprehend our meaning and understand how this sense of stability helps to give to the moral cognition a finality-value which other experiences cannot have.

The so-called "universality" of the Moral Imperative is due to its impersonality and to the supposition that the dictates of the moral law are, or should be, universally valid. Concerning the latter we have nothing to say; to-day, the relativity of the moral conscience and its freaks, need no expositor. With regard to the former—its impersonality—we have to ask how comes this experience to be separated, in common judgment, from the individual self, and to be looked upon as transcending it? What, in its nature, made it possible for Kant to lift it up above those cognitions that *belong* to the *individual?* On the ground of its universality, he might as well have classed the liking for salt together with it as impersonal and universal. To this query we would give the same answer as to the preceding one: the absence of sensations from the external senses, and especially the absence of sensations from the internal organs is, together with its involuntariness and its relative universality, the justification of the expression "impersonal" when applied to the moral imperative or to "reason." It is evident that the independence of this cognition of the will would not, of itself, be sufficient to warrant its classification as "impersonal;" otherwise simple sensations would be just as much impersonal.

The difficulty vanishes when we realize that the great test of that part of our experiences usually called the "self" is, in non-voluntary experiences, pungency and localizability. Of two non-voluntary experiences, the one having the greatest sensational and emotional vividness and the most definite bodily localization is the one that we feel to be most our "own." The lessons of multiple personalities and anæsthesia are full of instruction on this point. Those which above all else seem most certainly ours in this narrow sense, are the sensations of touch, those of motion and those arising from the activity of the organs of the vegetative life. The latter form the great massive substance of the cœnesthesic self in which our supra-sensory cognitions, *i. e.*, our thoughts and judgments, are imbedded. It constitutes the incarnated self; everything is all right when our higher experiences are permeated by it, but let this background fall out and the self becomes a shadowy remembrance. Now, according to our theory, the Moral Imperative is conditioned by a reflective purely cerebro-spinal process, *i. e.*, it contains neither direct sensations from the external, nor sensations from the internal organs. Consequently it must feel as a disincarnated experience, as a disembodied unlocalized manifestation of spiritual life.

As regards the imperativeness of the phenomenon before us, we need not add much to what has already been said to make the assigned physiological counterpart satisfactory in this respect also. When an afferent process discharges through well defined motor paths, towards muscles coördinated for a specific action, we feel impelled to perform that action; if the process, for any reason whatsoever, recurs frequently, we have an insistent impulse which may well be called a non-moral imperative; and if such an imperative is the motor conclusion of a reflective process, free from sensations from the vegetative life, we have a moral imperative. It is moral because it follows upon a non-arrested, completed, reflective process, and is, on that account, accepted as final, approved as binding upon the subject. We are thus not only commanded to do a particular thing, but we also approve of it, and accept it; hence, its obligatoriness, its categoricalness. In the analysis of some of our illustrations we found reflex non-sympathetic processes that were imperious, but not morally imperative. In those cases the imperative was negated by the reflective process which came after, *i. e.*, in final resort, we did not approve the reflex imperative. *The motor conclusions of a reflective, non-sympathetic imperative ideo-motor experience are always approved of as final:* in the nature of the case, it cannot be otherwise, as we shall see presently.

"It is unqualifiedly true," says Prof. James, in his "Principles of Psychology," Vol. II, p. 568, "that if any thought does fill the mind exclusively, such filling is consent. . . . But it is not true that the thought need fill the mind exclusively for consent to be there." We cannot here attempt a discussion of this very difficult question of consent, but one thing we must try and make clear since our subject requires it. There are many kinds of "consent" of the filling-presence type. While I have in mind the first process of illustration No. I (the "watch" imperative idea), I may be said to consent to it, but the consent belonging to the reflective process of the same experience does not belong to the same family of consents. The former is an automatic, but the latter is a reflective approval, *i. e.*, an approval from a higher "self." A subdivision must, moreover, be made in the reflective consents according as the process having the consent *quale* does or not include elements from the vaso-motor system. We have, therefore, three different species of approvals representing not all the meanings the word assumes, but the most important ones. Each one has its particular grade of value: the reflex consent stands at the foot of the scale of values; we think better of the reflective-sympathetic consent, and for the reflective cerebro-spinal kind of consent we have a superlative regard; we may not know why it is so, but that it is so is the experience of everyone.[1] It is a sort of Supreme Court; its judgments are final and unrevisable, except by itself. Experience has taught us that the consents of the first class are liable to be withdrawn by the higher, reflective processes, and also that no judgment is final until it has been pronounced by the reflective cerebro-spinal court. These judgments of worth have the same origin as those which place sensations at the bottom and the higher mental processes on top; the same as those also which make us attribute greater worth to intelligence than to brute physical force.

Because this supreme approbation *quale* belongs only to experiences dignified by a sense of impersonality, it was granted the similar honor of being looked upon as a suprapersonal, a divine approbation. *The imperativeness of the moral law is distinguished from the non-moral imperativeness by this highest kind of approbation accompanying its commands.*

Nothing need be added to what has already been said in explanation of the inefficacy of the Moral Imperative.

And now, when we put all these peculiar qualities to-

[1] For some explanatory hints we refer the reader to pp. 544 and 546.

gether, when we reflect that the Moral Imperative lacks the "feel" of the incarnated self, that its dictates are well nigh universal in the same society; that it comes and goes independently of our will; that its imperativeness is stamped with supreme, with categorical, approval, and yet does not coerce; that it utters its sentences in the quiet of relative passivity and is felt in antagonism to the fiery imperatives of the incarnated self, as well as in opposition to the blind promptings of reflex actions,—we wonder no more at the place assigned to it in the psychological schemes of philosophers who lived before, or who are ignorant of, the portentous reconstruction of the personality-concept at the hands of modern psychology and of the increased knowledge regarding the relation of mind and body.[1] They could not see the close bond of parentage existing between the Moral Imperative and the imperiousness of the vegetative life; still less could they understand the physiological mechanism underlying the various moral experiences and how it differs from that of non-moral cognitions. In the circumstances in which they were, the theories and concepts they evolved were the only ones apparently able to justify the psychic characteristics we have just reviewed.

The Moral Imperative, when looked upon as the psychic expression of a well-defined species of reflex-arc process, takes its natural and legitimate place in the unified psychic system of modern science. Thus viewed, it becomes the correlate of the latest and highest biological differentiation, since it requires, as a condition of its existence, the independence of the cerebro-spinal from the sympathetic nervous system.

This functional separation of the life of relation from the vegetative life marks the most basal and most important differentiation in the course of achievement in biological evolution. One need not be an anatomist or a physiologist to know that in the continuous "effort" of the organism to adjust itself to its environment, the life of relation has tended to become more and more independent of that of nutrition and reproduction. In the lower forms of animate beings,

[1] How quaint and obsolete seems the Kantian phraseology when looked at from the modern point of view. See, for instance, the characteristic passage ending with: "Here, then, we see philosophy brought to a critical position, since it has to be firmly fixed, notwithstanding that it has nothing to support it either in heaven or earth. Here it must show its purity as absolute dictator of its own laws, not the herald of those which are whispered to it by an implanted sense or who knows what tutelary nature."—"Fundamental Principles of the Metaphysics of Morals," tr. by Tho. Kingsmill Abbott, M. A., pp. 61 and 62.

where there is no specialized nervous system, the whole body responds to each and every stimulus. In some of the mollusks we find already two distinct nervous systems, considerably specialized, but still with large masses of nervous substance in common, in which stimuli of various origin unite and diffuse, so that such organisms cannot have experiences belonging exclusively to the life of relation.[1] In the grown man, the two systems, although inter-related, are largely independent; the visceral stimuli are largely confined to fixed channels, and there are numberless ideo-motor cerebro-spinal tracts, practically closed—so far, at least, as consciousness is concerned,—along which stimuli can travel without diffusing to a consciously appreciable extent in the sympathetic nervous system. In consequence of this high differentiation of functions, the various distinct types of processes Nos. II and IV are possible.

Individuals of different races, and those of the same race, differ widely with regard to emotionality and sentimentality. The recent neurological discoveries which show that the medullation of nervous fibres is far from complete at birth, but continues probably until late into middle life,[2] makes it easy to account on this ground for the general "emotional" difference existing between childhood and maturity. We may expect that a still more minute knowledge of neurology will bring to light the differences between races and sexes and individuals generally in the extent and completeness of the separation of the columns of fibres continuing the sympathetic system in the medulla and thence to the cortex of the cerebrum and cerebellum. Without stepping beyond legitimate deductions, we may well say that a history of the differentiation and isolation of the two nervous systems in man would give us, from the anatomo-physiological side, a parallel of the history of the antecedents, genesis and growth, of the moral sense, for reasons similar to those that show the anatomo-physiological history of the sense-organs to have moved step by step with the history of the "development" of the sensations.

Before closing we shall ask the readers' indulgence for a glittering generality, well worth considering, however. If the "endeavor" of the organism through its career of evolution is shown by comparative anatomy to have been towards the isolation of the sympathetic from the cerebro-spinal system, the greatest and most portentous *conscious* effort of the highest races during past millenniums has been to deliver the

[1] See plates in Gustav Retzius' "Biologische Untersuchungen."
[2] See especially Flechsig's work.

"soul" from the influences of the "body." Who has not been deeply impressed by the fury, or at least the penetrating intensity, with which the prophets of the ethico-religious consciousness of the Indo-European races have preached the subjugation, if not the annihilation of the body, abdication of self, renunciation; *i. e.*, indifference to humiliations, to the wounds of vanity and the cravings and pains of the flesh, that we may walk in the pure light of the uncarnated spirit? Hindoo, stoic and Christian sages have done battle under the same banner. It would be easy, but it is unnecessary, to quote in support of this statement the sayings of Gautama, of his disciples, those of Jesus or his immediate disciples, and especially of the saints of the middle ages. Translated into modern language this baleful "flesh" or "body" stands for the experiences dependent upon the sympathetic nervous system; it is the manifestations of that part of the self roughly denominated "the vegetative life," while the "soul" designates in the philosophy of the Church, as far as it stands for conscious realities, the experiences dependent upon the cerebro-spinal nervous system: it is the life of relation. And so it appears that the crusade of the ethico-religious consciousness, is a war of the cerebro-spinal Self against the cerebro-sympathetic Self: a war recorded not only in the literary annals of humanity, but also, and in a more lasting manner, in the neuro-physiological mechanism of the survivors of the Struggle for Life.

The common man is struck with a momentary joyful amazement when it dawns upon him that the deliverance after which humanity has been and is still yearning, is the self-same deliverance towards which these ill-treated bodies of ours have tended, and which they have already achieved in a large measure.

The biological conception of the Moral Imperative developed in these pages appears to us as one of the stones of the psycho-physiological foundation yet to come, upon which a truly scientific system of applied Ethics is to be built in the place of the practically useless "criterions of conduct" given us by ethical philosophers as the chief conclusion of their painstaking labors. When scientific answers have been given to the following queries: How are these Moral Imperative processes established? How is the passage made from the inefficacious command to the action?—a question involving the treatment of the will, in as far as it is a factor in the determination of action; and of algedonics, if it is to be considered in this connection. What is the value of Moral Imperatives, measured in their consequences with reference to pain and pleasure, or to character, or something else?—a

question upon which much light will be thrown by the investigation of the formation of the afferent side of the reflective arcs. When these and other questions have received a psycho-physiological answer, applied ethics will be in position to give the scientific instruction that humanity may rationally expect from it, but which it has thus far received only as the result of desultory empirical knowledge; *viz.*, methods of establishing clear moral imperative processes, of value, and of developing efficient motor connections between the knowledge of the dictates and their execution.

If the analysis of the Moral Imperative which has been made in this paper is correct, it obviously justifies a most far-reaching change of current conceptions with regard to the origin and nature of moral evil, of sin—and consequently of Regeneration—and of the relation of morality to belief in God and to many of the dogmas of theology.

DISCUSSION.

Professor Baldwin's Method of Studying the Color-Perception of Children.

A theory may be right or wrong, no matter whether there are many or few facts in support of it. On the other hand, a theory is right only when it can stand the brunt of a very large number of facts. If there are in the second case a few outlying 'facts,' the theory may still be right, because these facts may have been badly observed. If the facts on which the theory is based are very few or are equivocal, *i. e.*, differently interpreted by different observers, then the theory, though it may serve a man much in his own work, can take rank only as an individual hypothesis.

Professor Baldwin has lately published several theories in the domain of individual and child psychology. All of these theories have succeeded in getting themselves discussed; but no one of them, so far as I am aware, has got itself generally accepted. The general attitude of criticism seems to be that the theories are ingenious, and of the kind that would *a priori* carry conviction. But for this very reason they may be especially dangerous: first, because the uncritical mind may take for explanation what is only a working hypothesis; and secondly, because their schematic and suggestive exposition will make it very difficult for detailed criticism to reach them, the author being able to shift his ground and to modify his formulation at every attack. Still criticism must be passed, if only to check unconsidered acceptance; and serves its purpose if it rouses the author of the theory to a clearer, more definite and more comprehensive statement of his position.

Perhaps no theories are more likely to obtain credence than those which purport to lay a foundation for the science of education. While there are among educationalists some really careful thinkers, there are on the other hand, as one would naturally expect, thousands of industrious, thoroughly earnest and altogether untrained enthusiasts who have neither time, education, nor inclination carefully to weigh the multi-

tude of theories which are offered them, and which they eagerly desire to apply. Hence the fact that a psychologist in good standing has formulated and published a theory is not infrequently considered a sufficient evidence of its truth. Resulting from this comes a tendency to ignore all consideration of the theory itself, and to attempt at once its verification. But if theories are to be advanced with the abandon suggested by Prof. Baldwin, who says: "Give us theories, always theories! Let every man who has a theory pronounce his theory[1]!"—then surely the authors of these wholesale productions must expect them to receive especially critical examination, as theories, before their application is thought of.

Among other views that, it seems to me, must be expressed by his readers and considered by the author, before it can place itself on a practical working basis, I submit some of my own concerning Prof. Baldwin's theory of the law of suggestion or dynamogenesis as demonstrated in his experiments by the distance method upon the color perception of his child. They are the result of an honest study, undertaken primarily with the object of understanding the theory and its method.

The law of nervous dynamogenesis is briefly expressed in the statement that every state of consciousness tends to realize itself in an appropriate muscular development.[2] Suggestion in general is the tendency of a sensory or ideal state to be followed by a motor state.[3]

Prof. Baldwin has believed this for some time, and upon it has based the theory that the development of mind can be profitably studied. In 1890-1893 he published a series of articles in *Science* reporting observations on infants, and more recently (1895) in his book called "Mental Development in the Child and the Race: Methods and Processes," has gathered these together and attempted to synthetize his theory with that of the "current biological theory" of organic development. Along with the theory Prof. Baldwin has developed its application. In Chapter II of this book he explains his new method of child study, which is exceedingly attractive and very simple. He uses the color question to make clear his method, and says in this connection: "Only when we catch the motor response or a direct reflex, in its simplicity, is it a true index of the sensory stimulus in its simplicity." He thinks that hand movements are the most

[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 38.

[2] "Handbook of Psychology: Feeling and Will," p. 281.

[3] "Distance and Color-Perception by Infants," *Science*, XXI, pp. 231, 232.

nearly ideal in this respect. Sensations, he says, are stimuli to movement; and the child's efforts with its hands become indications of the relative degree of discrimination, attractiveness, etc., of the different sensations which call the efforts out.[1] Among others mentioned he thinks that the following questions might be taken up by this method:

(1) "The presence of different color sensations, as shown by the number and persistence of the child's efforts to grasp the color.

(2) The relative attractiveness of different colors, measured in the same way.

(3) The relative attractiveness of different color combinations.

(4) The relative exactness of distance estimation, as shown by the child's efforts to reach over distances for objects,"[2] etc., etc.

The first investigator to attempt any systematic experiments on children as to color sensation was Prof. Preyer, who, in his well-known work *Die Seele des Kindes*, published in 1881, gives the results of 1,486 judgments of color made by his child. The experiments began at the end of the second, and continued almost to the end of the third year. Prof. Preyer's problem was to ascertain what colors the child could distinguish and rightly name. It was an investigation of color discrimination. His method involved knowledge by the child of the names of the colors. This knowledge Prof. Preyer gave him by showing him a few colors and teaching him their names. Then with the colors still before the child, he asked him: Where is the red? Where is the green? etc., requiring the child to point out the color. Or—and this method he seems to have followed for the most part—Prof. Preyer would say: What color is this? requiring the child to name the color. From time to time during the course of the experiments new colors were added. At the end of the 34th month the results were as follows[3]:

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[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 45.

[2] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., pp. 45, 46.

[3] *Die Seele des Kindes*, 4th ed., p. 121.

	Judgments.		Per Cents.	
	Right.	Wrong.	Right.	Wrong.
1. Yellow	232	8	96.7	3.3
2. Brown	79	8	90.8	9.2
3. Red	235	36	86.7	13.3
4. Violet	139	24	85.3	14.7
5. Black	39	7	84.8	15.2
6. Rose	76	29	72.4	27.6
7. Orange	47	23	67.1	32.9
8. Gray	35	33	51.5	48.5
9. Green	101	123	45.0	55.0
10. Blue	61	151	28.8	71.2
Total	1044	442	70.3	29.7

The next investigator to produce a new method was M. Alfred Binet. It is fitting, however, to speak, in the first place, of the experiments of Miss Milicent Shinn, who followed in the main Prof. Preyer's method, experimenting upon her niece in the third quarter of the child's second year. Here are her results:[1]

	Judgments.		Per Cent.	
	Right.	Wrong.	Right.	Wrong.
1. Pink	35	0	100	0
2. Orange	170	1	99.4	.6
3. Black	108	1	99.1	.9
4. Green	311	8	97.5	2.5
5. Yellow	240	7	97.2	2.8
6. Blue	309	22	93.4	6.6
7. Brown	11	1	91.7	8.3
8. White	117	11	91.4	8.6
9. Violet	63	10	86.3	13.7
10. Red	76	52	59.4	40.6
Total	1440	113	92.7	7.3

[1] "Notes on the Development of a Child" (University of California Studies), p. 49.

Prof. Preyer's child showed a tendency to confuse green and blue, which consequently appear at the end of his list; while the little girl subject of Miss Shinn became early uncertain regarding blue and red, with the result that red is tenth on her list. Yellow Preyer's child liked and discriminated best; and though it ranks fifth on her list, Miss Shinn is inclined to think it the favorite color of her niece. A comparison of the two tables, however, does not enlighten us to any great extent as to the probable tendency of color discrimination in young children. Much of the variation is probably due to word confusion.

M. Binet, realizing this, and also realizing that colors might be distinguished before word association is established, tried to obviate the difficulty by what he called the *méthode de reconnaissance.*[1] Preyer's method he calls the *méthode d'appellation.* Prof. Baldwin says that Binet, under the *méthode d'appellation*, "varied the conditions by naming a color and requiring the child to pick out the corresponding color," intimating that this was a new departure. Indeed, he says that "this gave results different not only from Preyer's, but also from those which Binet reached by Preyer's method."[2] But Preyer's method included *both* the naming of the color by the child *and* the pointing out of the color which was named. Sometimes he took the one way, sometimes the other. Binet's departure was the separation of the two. After carrying on both processes side by side for a while, as Preyer himself had done, Binet separated them, and kept a record of each. In one series he required the child to name the color; in the other to point out the color named. He made altogether by the *méthode d'appellation* 508 tests, which, arranged in percentages of right judgments, average as follows: red, 99; blue, 96; orange, 93; maroon, 86; rose, 74; violet, 71; green, 68; white, 54; yellow, 46. These results are obtained by averaging according to the number of tests given. Since these varied greatly (from 135 for red to 15 for orange), it would, perhaps, be fairer to find the percentage of right judgments in each of the three ways employed; *i. e.*, first, before the processes were separated; secondly, when the child named the color; thirdly, when she pointed to the color named: and to average these several results. The figures then are slightly different. Blue equals red and violet equals green; otherwise there is no change: red and blue, 99; orange, 93; maroon, 86; rose, 74; violet and green, 71; white, 54; yellow, 46. Binet's experiments

[1] *Revue philosophique*, Vol. XXX, pp. 583 ff.

[2] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 40.

were made upon his little girl, beginning when she was 32 months old, and continuing for a little over 6 months.

These, then, are the three tests made by Preyer's method, and they all differ widely. Yellow, so readily distinguished by Preyer's child, is by Binet's the least readily recognized. Red and blue, confused by Miss Shinn's niece, give almost no difficulty to Binet's little girl.

The results obtained by dividing Preyer's method are also interesting. When Binet's child said the name of the color, the order in percentages of right judgments is this: orange and blue, 100; red, 97; green, 83; maroon, 75; violet, 70; rose, 64; white, 33; yellow, 30. When the child indicated the color, the name being given her, the colors range themselves in this series: red, 100; maroon, 94; blue, 92; rose, 89; orange, 86; violet, 71; white, 68; green, 66; yellow, 58.[1] Binet by this clearly proves that the word element does affect the child's judgment. In order to test this still more closely, he carried on a series of experiments at the same time as the others by what he calls the *méthode de reconnaissance*. "I wanted to see," he says, "if this same child could find a color, which, after being shown to her, was mixed with several others. . . . . The *méthode d'appellation* places the child in rather artificial conditions. It is obliged to learn the names of the colors; it is forced to form a rational perception. It is evident that the child left to itself does not recognize colors by their names, but by visual memory; and the *méthode de reconnaissance* places it in more natural conditions than the other method."[2]

Prof. Baldwin, in discussing these methods, says: "It is, perhaps, a confirmation of Lehmann's position[3] that the

[1] In all calculations, .5 per cent. or more I have counted as 1 per cent.

[2] *Revue philosophique*, Vol. XXX, p. 589.

[3] Dr. Lehmann has an article in the *Philosophische Studien*, Vol. V, pp. 96-150, in which he shows that various grays upon rotating discs are recognized, if given names. Prof. Baldwin, in the "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 42, speaks of Lehmann as using "colored wools;" but gives no reference. In his "Hand-book of Psychology: Senses and Intellect," p. 177, he says: "As soon as some sign is made of a peculiar kind in connection with the image, it is recognized. Recent experiments by Lehmann on the recognition of differences of color strikingly confirm this view." The reader is referred to Lehmann, *Philosophische Studien*, Vol. VI. But Lehmann has no article in that volume. The volume referred to is probably V, in which appears Lehmann's *Ueber Wiedererkennen*. Dr. Burnham, in the AMERICAN JOURNAL OF PSYCHOLOGY, Vol. II, p. 610, referring to this article, says that Lehmann experimented with sensations of color, using different shades of gray produced by means of rotating discs. Both Prof. Baldwin and Dr. Burnham speak of Lehmann as

colors least recognized in Binet's list are shades whose names are less familiar to children; his list in order of certainty of recognition is red, blue, green, rose, maroon, violet and yellow, by the *méthode d'appellation*; and by both methods together, red, blue, orange, maroon, rose, violet, green, white and yellow."[1] In a footnote he says that the calculations were made from Binet's detailed results (*Revue philosophique*, 1890, pp. 582 ff.) by Mr. Tracy, and cites the latter's book, "The Psychology of Childhood."

It is not easy to see how Prof. Baldwin gets these results from Mr. Tracy's figures. For example, Mr. Tracy does not give any computations for the *méthode de reconnaissance*; and even if he had, there could have been no orange in the list of colors, as Binet did not make use of orange in the tests by this method. Prof. Baldwin, as will be seen, gets a certain order for the *méthode d'appellation*, in which orange is wrongly omitted: Mr. Tracy is probably responsible for this. But in the series Prof. Baldwin gives of both methods together, *i. e.*, the *méthode d'appellation* and the *méthode de reconnaissance*, the orange appears. Where he gets it, is not apparent. Mr. Tracy himself has interpreted Binet's tables for the *méthode d'appellation* very curiously, sub-dividing each series into a first and second series. The tables show no warrant for this. Mr. Tracy's results, therefore, wrought out in this peculiar manner, are confusing.

Prof. Baldwin criticises all of these methods and points out that even Binet's *méthode de reconnaissance* does not get rid of word association, since probably those colors are most easily recognized whose names are best known. It is here that he cites Binet's list. Taking the series that Prof. Baldwin gives of both methods together: red, blue, orange, maroon, rose, violet, green, white and yellow,—if we share his views we shall have to say that white is a less familiar name to a child than violet, and that green is a stranger word to him than maroon.

No other psychologist, I think, will agree with Prof. Baldwin in his position on the several points just discussed; but, on the other hand, none will fail to agree with him as to the general inadequacy of the methods so far advanced for the

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experimenting with colors; whereas, in reality, he worked only with *grays*. It may be that Prof. Baldwin, having made this first mistake, slipped naturally into the second one,—the use by Lehmann of wools. Neither in *Ueber Wiedererkennen*, nor in anything else that Lehmann has written, can I find any evidence of his having experimented with colored wools.

[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 42.

investigation of color perception in children, and the need for a better one.

Prof. Baldwin now comes forward with a new method, based on the law of dynamogenesis referred to above.[1] He thinks that the young child's reaching movements reflect its sensibility. The two variables to be considered in this method are the quality of the stimulus and its distance from the child. The drawing-out influence of the stimulus will vary with the quality and in inverse ratio with the distance of the colored object. While Prof. Preyer's experiments were directed upon color discrimination, and M. Binet's by the *méthode de reconnaissance* upon color recognition, Prof. Baldwin says: "I undertook at the beginning of my child H.'s ninth month to experiment with her with a view to arriving at the exact state of her color perception, employing this new method." Bits of colored blotting paper were placed at different distances from the child, one at a time. Account was kept of the number of times she reached out toward each. The colors were blue, red, white, green and brown. Newspaper (presumably cut to the same shape and size as the colored pieces of blotting paper) was used as "a relatively neutral object." Yellow unfortunately was not used, as Prof. Baldwin could not obtain a yellow in his neighborhood that suited him. Since the experiments extended over a period of six months, it is difficult to understand why he did not have recourse to some other neighborhood. Below are the tables showing the results of Prof. Baldwin's experiments, as they appear in the second edition of "Mental Development in the Child and the Race: Methods and Processes."[2]

R is the symbol for refusal, A for acceptance. N stands for the entire number of experiments with each color respectively, and n for the entire number with all the colors at each distance respectively.

$\frac{A}{N}$ = the proportion of acceptances or efforts for any color, and

$\frac{R}{n}$ = the proposition of refusals for each distance.[3]

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[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., Chap. II.

[2] See p. 52.

[3] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 54.

TABLE I.

DISTANCE, INCHES.	9	10	11	12	13	14	15	TOTALS.	RATIO $\frac{A}{N}$
	R. A.	R. A.	R. A.	R. A.	R. A.	R. A.	R. A.	R. A. N.	
Blue	0—1	0— 4	0—5	1— 3	2— 4	1— 5	3—1	7— 23— 30	.766
Red	0—1	0— 3	2—2	1— 4	1— 7	1— 7	5—1	10— 25— 35	.714
White	0—0	0— 0	0—0	0— 1	0— 5	1— 1	3—0	4— 7— 11	.633
Green	0—0	0— 1	0—1	2— 1	1— 4	1— 2	2—0	6— 9— 16	.60
Brown	0—1	0— 2	2—1	3— 2	0— 3	3— 1	2—0	10— 10— 20	.50
Totals	0—3	0—10	4—9	7—11	4—23	7—16	15—2	37— 74—111	.66
Ratio $\frac{R}{n}$	0	0	.30	.39	.15	.30	.90	Total, .33	

TABLE II.

DISTANCE, INCHES.	9	10	11	12	13	14	15	TOTALS.	RATIO $\frac{A}{N}$
	R. A.	R. A.	R. A.	R. A.	R. A.	R. A.	R. A.	R. A. N.	
Newspaper				0—17	0—28	1—33	25—2	26— 80—106	.76
Color	0—3	0—10	4—9	7—11	4—23	7—16	15—2	37— 74—111	.66
Totals	0—3	0—10	4—9	7—28	4—51	8—49	40—4	63—154—217	.71
Ratio $\frac{R}{n}$			.30	.20	.07	.14	.91	Total, .29	

Despite the fact that this is the third time of their printing, the tables still contain 7 errors in calculation. In Table I, under the sub-head *N* of column headed *Totals*, 16 should

read 15. Under *Ratio* $\frac{A}{N}$ for green .633 should read .636. *Ratio* $\frac{R}{n}$ for 11 inches should be .307, not .30. (It ought consistently to read .31, since in the next column .388 is made .39, and in that following .148 is put down as 1.5). Under 15 inches *Ratio* $\frac{R}{n}$ should be .882, not .90. In Table II *Ratio* $\frac{R}{n}$ for 11 inches, .30 again should read .307 or .31, not .30. *Ratio* $\frac{A}{N}$ for newspaper should be .754, not .76, and for colors .666, not 66.

As first printed in *Science*, April, 1893 (leaving out of account some less considerable inaccuracies of fractional percentages), the tables contain the following errors: In Table I under the sub-head *R* in the column *Totals* for green, 7 should be 6; under *N* 16 should be 15. Under the same headings for brown 11 should be 10, and 21 should be 20. Under column *Ratio* $\frac{R}{N}$ for green, .56$\frac{1}{3}$ should be .60, and for brown .47$\frac{1}{2}$ should be .50. In Table II *Ratio* $\frac{R}{n}$ for 11 inches should be .307, not .33$\frac{1}{3}$, and for 15 inches *Ratio* $\frac{R}{n}$ should be .882, not .89.

As printed in the first edition of "Mental Development in the Child and the Race: Methods and Processes," the tables contain 16 mistakes, ranging in error of percentage from 2 to 15. In the preface to the second edition, the author says: "The demand for a new edition of my book gives me the opportunity to make certain minor corrections throughout. The only important alteration is to be found in the tables (I and II) on p. 52, in which certain columns had been substituted from other tables which lie unpublished among my papers." Nevertheless, recalculation has disclosed the seven errors referred to above.

These facts are significant in themselves: for a writer who is careless in his presentment of facts we shall incline to suspect of being careless in his interpretation. Indeed, Prof. Baldwin's interpretation of these tables is as puzzling as the tables themselves. I have been able to understand it only on the assumption that *some of his statements in the second edition are based upon the figures of the first edition* of his book.

(1) His first statement is that "the colors range themselves in an order of attractiveness, *i. e.*, blue, white, red, green and brown." While this is a possible interpreta-

tion of the tables of the first edition, a glance at column *Ratio* $\frac{A}{N}$ shows that red should rank next to blue. In the *Science* article, "Distance and Color Perception by Infants," Apr., 1893, the interpretation is given correctly. It would seem, then, that Prof. Baldwin interpreted from the 'substituted' tables of the first edition, where *Ratio* $\frac{A}{N}$ reads: blue, .78; red, .75; white, .78; green, .68; brown, .50.

(2) White, he says, was more attractive than green, and slightly more so than red. But, according to the figures of the second edition, white is .08 *less* attractive than red. This misstatement also may have been caused by reading from the incorrect table.

(3) The next sentence says: "The newspaper was at reaching distance (9-10 inches) and a little more (up to 14 inches) as attractive as the average of the colors, and even as much so as the red."[1] We *are not given the figures* for newspaper at 9, 10 and 11 inches; but beginning at 12 inches and going up to and including 14 inches, we find that newspaper is *far more attractive* than any color tested, even than blue. The proof is obvious: *Ratio* $\frac{R}{n}$ (proportion of refusals) at 14 inches, for blue is .166, for red is .15, for white is .50, for green is .33⅓, for brown is .75, and for newspaper is .029; while *Ratio* $\frac{R}{n}$ for newspaper at 12 and 13 inches is 0.

(4) The attractiveness of the newspaper in these distances Prof. Baldwin accounts for by the fact that "the newspaper experiments came after a good deal of practice in reaching after colors, and a more exact association between the stimulus and its distance." Reaching distance, we are told, was 9 to 10 inches. If the child was improving in her estimation of distance, one would expect her, at a distance of 14 inches, to begin to refuse the newspaper, simply because she knew that she could not reach it; yet she does so in only 1 case out of 34, while the colors are refused at 14 inches in 7 out of 16 cases. The explanation, too, that these newspaper experiments came after a "good deal" of practice in reaching, is not altogether satisfying. Since a careful record was kept of each test, it would have been easy for Prof. Baldwin accurately to state just how much practice the child had received before the newspaper stimulus was offered her. Failure to state the conditions of his experiment on the part of an investigator must result in

[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 53.

failure to accept his conclusions on the part of his readers. Prof. Baldwin could and should have given this condition.

(5) While the *attractiveness* of the newspaper up to 14 inches is accounted for by the child's practice in reaching, the *refusal* of the newspaper at 15 inches is accounted for, curiously enough, in the same way. "At 15 inches and over, accordingly, the newspaper was refused in more than 92 per cent. of the cases, while blue was refused at that distance in only 75 per cent., and red in only 83 per cent." Since the child had had "a good deal of practice" in reaching, and had established a more exact association between the stimulus and its distance, it is hard to account for the use of the word 'accordingly' in the above quotation. One might better attribute the refusal of newspaper at this greater distance to the dynamogenic power of color (and this Prof. Baldwin seems to do in citing the smaller percentages of refusals of blue and red), were it not for the fact that green, white and brown, respectively, are refused at this distance in 100 per cent. of the cases!

(6) Prof. Baldwin then calls attention to the striking fact that the child refused persistently to reach for anything put at 16 inches or more away from her, and at 15 inches refused 91 per cent. of all the cases, 90 per cent. (the correct figures are 88.2 per cent.) of the color cases, and 92 per cent. of the newspaper cases. This shows, he thinks, the very accurate visual estimation of distance acquired by the child. "The child's interpretation of the distance inhibits all effort to reach across it." These interpretations result, we are told, from "associations of visual indications of distance with sensations of hand and arm movement."[1]

The inclination to accept this result and its explanation, were there no other reasons, would be destroyed by Prof. Baldwin's failure to state conditions and his vague use of terms. For example, after giving on page 54 the above figures for distances of 15 inches and over (the explanation for them being presented on pp. 76-77),[2] Prof. Baldwin says: "At nearer distances we find the remarkable uniformity with which the *safe-distance* association works at this early age. At 14 inches only 14 per cent. of all the cases were refused, and at 13 inches only about 7 per cent." The natural inference here is that 14 inches is included in the '*safe-distance*.' But on page 77 he speaks of the *safe-reaching dis-*

[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., pp. 76-77.

[2] The habit of later incidental explanation is characteristic of the book, and is very confusing.

tance; the *uncertain-reaching distance*, and the *impossible-to-reach* distance; and on page 49 he says that "in some cases the inhibition of *d* (distance) does not work, and the child oversteps all its experience in violent straining and tears." On page 53 he calls reaching distance 9 to 10 inches; and on page 54, again, says that there were no refusals to reach for anything exposed within reaching distance (10 inches). Such, then, are the various statements for us to attempt to reconcile.

If we include 14 inches within the *safe-reaching distance*, and put 15 inches in the category of *impossible-to-reach distance*, then *uncertain-to-reach distance* is excluded. Possibly 9 to 10 inches is intended to mean what we may term '*easy-reaching distance*,' and 14 inches that distance which calls forth "straining and tears;" but this does not explain *safe-distance*. If 9 to 10 inches is the distance beyond which the child cannot reach, and if this is also the *safe-distance*, then it is hard to see how the *safe-distance* association worked, as we are told it did, at 14 inches. If, on the other hand, 14 inches is the 'strain' limit of her reaching distance, and is included in the *safe-distance*, not only does Prof. Baldwin's *uncertain-reaching* distance disappear, but his use of the term "reaching distance" in other connections cannot be understood.

Leaving us in a state of uncertainty as to what he means by *safe-distance* association, Prof. Baldwin goes on to speak of the results obtained with objects displayed within 10 inches. He says: "The fact that there were no refusals to reach for anything exposed within reaching distance (10 inches)—other attractive objects being kept away—shows two things: (1) the very fine estimation visually of the distance represented by the arm-length, thus emphasizing the element of muscular sensations of arm-movement in the perception of distance generally; and (2) the great uniformity at this age of the phenomenon of 'sensori-motor suggestion' upon which this method of child study is based." Sensori-motor suggestion is here conditioned by three facts: (1) distance from the object, (2) its attainment or non-attainment, (3) its quality. That Prof. Baldwin is not unmindful of the second condition is shown by what he says in giving the formula of the dynamogenic method: "If the child reaches for a blue-*q* [q=quality] at 12 inches, and just gets it, he will then reach for a green-*q* with greater avidity at twelve inches than he would otherwise have reached for the same green-*q* at nine inches."[1] But

[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 48.

again we are left in the dark. We are not told whether the child at "reaching distance" really succeeded in grasping the object or not. How important this condition is, the above quotation plainly shows. The only information we are given relating to this point is on page 56, where Prof. Baldwin says: "Further, after each effort or two the child should be given the object reached for to hold or play with for a moment; otherwise he grows to apprehend that the whole affair is a case of Tantalus." "After each effort or two" is not as definite as the statement should have been made. Presumably, however, the child was given the object very frequently, whether she was able to reach it or not. In that case, condition (2) (attainment or non-attainment of the object) is almost, if not entirely, ruled out so far as the experiments are concerned, and the child's refusal to reach for objects at 15 inches and over is so much the more striking. On the other hand, any irregularity in giving the child the object would tend to affect her reaching. This would also influence her perception of color. Unless great care were taken that each color secured the privilege of being held or played with, "for a moment," an equal number of times, there would be danger that those colors receiving oftenest this closer contact would become the most readily recognised. Again, an object played with might be reached for, and yet not possess in its color quality enough dynamogenic power to call out movement.

(7) He says that the fact that there was a larger percentage of refusals at 11 and 12 inches than at 13 and 14 inches is due to the influence of brown, "which was consistently refused when more than 10 inches away." If brown was consistently refused when more than 10 inches away, then it must have been refused at 11, 12, 13, 14 and 15 inches respectively, and one cannot understand how that accounts for there being more refusals at 11 and 12 inches than at 13 and 14 inches. Clearly it does not. But even if this were a reason, it could not be advanced; for a glance at the tables shows that brown was *not* refused at 13 inches, but was accepted every time. If it had been refused as often at 13 inches as at 11 and 12 inches, the percentage of refusals for 13 inches would have been much higher. There is a valid reason, it seems to me, that Prof. Baldwin might have brought out, and that is the small number of tests given at 11 and 12 inches for green and white. The total number of tests for green at 11 and 12 inches is 4, for white 1; while the total number for green at 13 and 14 inches is 8, and for white is 7.

(8) Newspaper taken as a "neutral object"—and we are

not at all sure that it *is* neutral to the child—is not put on a fair basis of comparison. The tables give us no tests for the first three reaching lengths. If they did, newspaper would presumably rank ahead of the colors in attractiveness; for the nearer distances give a preponderance of acceptances. Ten out of the 23 acceptances for blue come within 9, 10 and 11 inches with no refusals; 6 out of the 25 for red with only 2 refusals; 2 out of the 9 for green with no refusals (green was not tested at 9 inches); and 4 out of the 10 for brown with 2 refusals. Putting it differently: at 9 and 10 inches there are no refusals for the colors, and at 11 inches the acceptances for the colors collectively are 70 per cent. It is safe to infer that had it been tested at this range the *Ratio* $\frac{A}{N}$ for newspaper would have exceeded even that of blue. There is one other way to place newspaper on an equal footing with the colors, and that is to exclude the first three distances. If we do so, the stimuli range themselves in the following order: newspaper, .754 (given in the table .76); red, .703; blue, .65; white, .636 (given in the table .633); green, .538; brown, .444. White, like newspaper, was not tested at all in the first three distances, and green but twice.

The relatively small number of tests made at the nearer distances, 9, 10 and 11 inches (none at all, as has been said, being given for white, only two for green and none for newspaper), the wide variation in number of tests (ranging from a totality of 3 experiments for all objects at 9 inches to 57 at 14 inches), added to the fact (which Prof. Baldwin himself recognises) of the absolutely small number of experiments represented by the tables,[1] forbid any accurate comparisons between stimulus and stimulus, or any safe conclusions upon the results as a whole. Yet Prof. Baldwin has not hesitated to make both.

In *Mind*, Apr., 1896, he says that he was not testing color discrimination or color preference, but color perception. It would seem, then, that he is illogical when he compares his results with those of Preyer and Binet, as he does on page 53 of his book. "Disregarding white, the difference between blue and red is very slight as compared with that between any other two. This confirms Binet as against Preyer, who puts blue last, and also fails to confirm Preyer in putting brown before red and green. Brown to my child—as tested in this way—seemed to be about as neutral as could well be." One would certainly infer from this that Prof. Baldwin

[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 57.

thought that the child not only perceived the colors, but *discriminated* between them; yet there is nothing in the experiments to tell us this. Moreover, Prof. Baldwin disclaims having investigated for discrimination. His comparisons, then, of his results with Preyer's, who did investigate for discrimination, are, to say the least, somewhat inconsistent.

He criticises Preyer because, he says, Preyer's results cannot be analysed; yet his own are open to the same objection. He tells us that he experimented with his child in order to arrive at "the exact state of her color perception."[1] The interpretation of the tables, instead of giving us any information about "color perception," begins with this statement: "The colors range themselves in an order of attractiveness." The natural inference is that Prof. Baldwin considers 'order of attractiveness' and 'order of perception' as synonymous expressions. But surely it does not necessarily follow that because the child reached out proportionately a greater number of times for blue than for red, she perceived blue better than red. It may have been that the child had a feeling for blue *plus* the sensation, and that to this feeling was due the greater 'attractiveness' of blue. Brown, on the other hand, may have produced in her a feeling of repulsion, or may not have been noticed as color at all, and the motor response have been called out simply by the form of the object.

Prof. Sully in *Mind*, Jan., 1896, reviews "Mental Development in the Child and the Race: Methods and Processes." He suggests that at first the objects grasped at be some sort of uncolored playthings, and then that they be colored and used as stimuli. In this way the last objection made would be to some extent avoided. The difficulty, however, would be in determining upon an 'uncolored' object. Prof. Sully also suggests that we do not know how soon children are biased in their preference by the colors of their environment, the mother's dress, for instance.[2]

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[1] "Mental Development in the Child and the Race: Methods and Processes," 2d ed., p. 50.

[2] Since writing this article I have found another reference to Prof. Baldwin's method in Prof. Sully's "Studies of Childhood," pp. 20-21. He repeats again the statement made in *Mind* that the method does not test color discrimination, but color preference, and adds that, even as a test of preference, it is likely to be misapplied: if the colors are not equally bright, if one color falls more often into the first or fresh period of the experiment, or if one color be brought in after longer intervals of time than another.

The first point has been discussed in this paper. Prof. Baldwin says he was not testing color discrimination. The necessity of equality of brightness he himself calls attention to on page 56 of his

The tables do not prove, again, that the child perceived blue, red, white, green and brown as separate colors, or that certain of these were (on account of their peculiar color qualities) more attractive than others; because newspaper, the 'neutral' object, even under its unfair conditions was as 'attractive' as any colored stimulus.

Prof. Baldwin's new dynamogenic method, then, ingenious as it is in its conception, and attractive as it is in its simplicity, has, nevertheless, been so carelessly carried out by its author that no reliance can be placed in the results obtained. True, the idea still remains; and it is left, possibly, for some less brilliant but more accurate experimenter to demonstrate its worth.

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book. As to how he met the fatigue factor, no statements are given except that experiments were stopped if the child showed signs of weariness. Whether or not Prof. Baldwin considered the attractive force of novelty which would result from the fact that one color is presented after a longer interval of time than another we do not know. As has been said, we are given no information as to the regularity or irregularity with which the various colors were presented.

BOOK NOTES.[1]

(G. S. H.)

De la Nature de l'Épilepsie. Dr. FR. HALLAGER. 1897, pp. 181.

Epilepsy is anæmia. Lack of nutrition is a potent irritation. The attack may be more or less sudden and intense, according as the supply of blood is slowly or suddenly shut off. Some centres may be more excitable or exhausted than others, and so react differently to the same stimulus. The vessels may be constricted generally or locally. External irritation, as of a tumor, syphilis, decayed teeth, may cause it in otherwise sound brains by acting on the vaso-motor system. Vaso-motor irritability is thus a function. Ideopathic epilepsy is due to latent lesions. Cortical is really reflex epilepsy, for irritability can rarely be so great as to produce the results with no irritation.

Grundriss der Psychiatrie. Dr. C. WERNICKE. 1894. Part I, pp. 80. Part II, pp. 97.

These interesting lectures base psycho-cerebral diseases on the distinction between projective and associative systems. Speech, vision, memory, organic somatic sensations, things and reality as a function of the projective system and spontaneous and reflex movements are described as neural functions. Ideas of movement and touch are originally organic sensations, consciousness, personality follow, and with the ninth lecture morbid psychology begins. Autopsychic, allopsychic, somatopsychic and autochthonous hallucinations are distinguished. Sejunction explains the disintegration of individuality. Phonemes are illusions of speech sounds. The illusion of retrospective explanation is described.

Anleitung beim Studien des Baues der nervösen Centralorgane im gesunden und kranken Zustande. Von HEINRICH OBERSTEINER. 1896, pp. 572.

This third edition is much enlarged and rewritten, and now has 205 cuts.

Le Sommeil. MARIE DE MANACÉINE. Translated from the Russian with the consent of the author by Ernest Jaubert. 1896, pp. 358.

This is a compendium on the pathology, physiology, hygiene and psychology of sleep and is provided with an excellent bibliography and index. It does not attempt new views, but is a very convenient and valuable *résumé*.

Hypnotism. ALBERT MOLL. 1897, pp. 448.

This is revised and enlarged from the fourth German edition. With its excellent index of subjects and of authors, its bibliography,

[1] Notice under this heading does not preclude a fuller review later.

and its treating of the recent development of hypnotism, it is now probably the best general survey of the subject.

L'Évolution des Idées Générales. TH. RIBOT. Paris, 1897, pp. 260.

Animals abstracted before words and their logic of images is first described. Infants, deaf-mutes and gestures illustrate higher pre-verbal forms. Animal language, classification, the evolution of number, space, time, cause and law are each given a chapter. The two chief causes of abstracting and generalizing are utility and invention, and the power unfolds in three directions—the practical, speculative and scientific. Conscious and unconscious faculties coöperate throughout.

Grundzüge der Psychologie. HERMANN EBBINGHAUS. 1897. Erster Halbband, pp. 320.

The name, origin and seat of the soul, its relation to the body, consciousness and unconsciousness are discussed in the first half, and the second is devoted to structure and function of the nervous system. The book is an introduction to the study of these topics and not merely a general compendium, and hence the author tells us is both full and detailed. It is not limited to experiment and result. As it is to cover the entire literature of the individual soul from its lowest to its highest experience, it will require at least three more volumes the size of this to complete it.

Animals at Work and Play. C. J. CORNISH. Second edition. London, 1897, pp. 323.

Animals' beds, sleep, toilet, society and dislike of solitude, etiquette, tactics, humor, grief, play, in pageants, in rain, soaring, lost in storms, what they see, industries, in sickness, longevity, migration, their sanctuaries, the iced Arctic food of far north migrating birds, the invisible food of fishes,—these and many other topics are described in a most charming and novel way. All is based on a very long and careful study of many animals.

Psychologie der Naturvölker. JACOB ROBINSOHN. Pp. 176.

This work is a series of nine ethnographic parallels on the discovery of the soul, anthropophagy, character of the dead, modes of burial, life of the soul after death, human offerings and retribution. Soul is conceived at first as heart, blood and breath, and each may have several souls. Bodies are eaten where soul and body are nearly or quite identified. The eater inherits the soul. Among savages the love of life is measured by the evil and consequent fear of the souls of the dead. Burial gives the best of all revelation of views concerning the dead. Burning, *e. g.*, frees the soul quickest. From consecrated ground open spirit ways to heaven. The bibliography of each chapter is excellent.

The God-Idea of the Ancients, or Sex and Religion. ELIZA BURT GAMBLE. 1897, pp. 339.

This is an amplification of ideas, most of which were hinted at in the author's "Evolution of Woman." It will be an unwelcome conclusion to most, should the world have to accept an hermaphroditic conception of deity, or to tolerate the view that the gross idea of sex has played so large a part in the evolution of primitive theology. The author seems to grudge deity his predominance of virile elements.

La Psychologie dans L'Opéra Français. LIONEL DAURIAC. 1897, pp. 162.

This is devoted to Auber, Rossini and Meyerbeer, as illustrating a stage in the evolution of French taste. The tragic, pathetic and picturesque elements in the opera of *William Tell* and the musical qualities and dramatic style in *Robert le Diable* are the chief topics.

St. Augustine et le Néo-Platonisme. L. GRANDGEORGE. Paris, 1896, pp. 158.

Neo-platonism caused Augustine to break with Manicheism, aided to bring him to Christianity, and gave him arguments against heresies, but he transformed neo-platonism on many important points. He always subordinated his philosophy to his faith where they conflicted. His doctrine of God owed most to it.

Je Pense, donc je Suis. PAUL VIALLET. Paris, 1897, pp. 140.

This is a treatise on philosophic doubt. The Cartesian principle is defended against current criticism that it is sterile, uncertain, and illegitimate. It is in fact both a syllogism and an intuition, and not only valid against materialism and skepticism, but opens one of the best lines of training for the young.

Preuve Philosophique de l'Existence de Dieu. FERDINAND DRION. 1896, pp. 52.

Proof of God's existence must rest on science, which describes his works. Matter cannot exist by itself, but one can never know why God made the world. Order implies design; the contingent, the necessary; the finite, the infinite.

La Modalité du Jugement. LÉON BRUNSCHVICG. Paris, 1897, pp. 246.

The modality of judgment is one of the essential problems of philosophy and is bound up with the very notions of intellectual activity. In judgments of both the practical and theoretical order, the soul affirms its interiority, even over against the self, and in comprehending has already resolved. Truth in affirmation carries truth in action, and the possibility of one is bound up in that of the other.

Die Grundsätze der modernen Weltanschauung. Dr. ADOLF BRODBECK. 1896, pp. 80.

Ancient ideas were dualistic, ours monistic. Both are presented in a few pages each. Phenomena once thought, as the name suggests, mere appearances, are now considered by inductive minds the basis of all. Ground and appearance, God and world, now coincide, hence deduction is obsolete. Evolution, materialism, liberty, equality and fraternity are the watchwords.

The Opposites of the Universe. MARIE SANDS. 1897. Third part, pp. 87.

This work discusses the theological and nomological opposites and is really a discourse about God. Opposites are chemical, astronomical, electric, zoölogical, geological, philological, religious, etc. Of all these, that between God and man is the chief. The world prayer of mankind is, perhaps, the best of this somewhat anomalous work.

Schopenhauer's System in its Philosophical Significance. WILLIAM CALDWELL, M. A., D. Sc. 1896, pp. 538. Charles Scribner's Sons.

This is a book of class lectures and is an excellent digest of Schopenhauer's theory of knowledge, of art, morals, religion and metaphysics, with a final general discussion of the system as a whole. The author corrects the impression that his pessimism is the leading trait of Schopenhauer's philosophy. It is a convenient and valuable presentation.

Histoire de la Philosophie. ÉLIE BLANC. 1896. Tome I, pp. 656. Tome II, pp. 660. Tome III, pp. 656.

The author is a professor of philosophy in the Catholic University of Lyons. A hundred pages are devoted to oriental pre-Greek systems, and it is full in the scholastic period, epitomizing many systems not usually treated in standard histories. The systems of the last half of the present century are given more than half of the last volume. The modern revival of scholasticism is fully brought out. The work is a monument of erudition.

A Manual of Ethics. Sir JOHN S. MACKENZIE, M. A. 1894, pp. 355.

The scope of ethics, its relation to other sciences, the moral judgment, duty, will and desire, happiness, perfection, freedom, the moral life, virtue, the inner life and higher individualism, evil, progress, relation to religion,—these are the topics of the interesting and vigorous work, here presented in a second revised edition. This work, which is in the University Tutorial Series, is the best ethical primer known to us.

Misère de la Philosophie. KARL MARX. 1896, pp. 291.

This reply to Proudhon's philosophy of misery,and here reprinted from the work of 1847, is a treatise on the metaphysic of social and political economy and an argument for free trade; and against the misery which theories of restriction have caused.

La Cause Première. EMIL FERRIÈRE. 1897, pp. 462.

Substance is the basis of all. Its manifestations as mother-energy are twofold, inorganic and organic. From the standpoint of truth substance is the first cause, from that of reality it is the world. It is not only primary, but necessary, absolute, eternal, infinite and universal. The world is contingent, limited, relative and singular. The normal conclusion we want,on the other hand,cannot be drawn. To attempt to infer them plunges us in irreconcilable contradictions. We must therefore accept, doubt and resign ourselves to ignorance at least provisionally.

Nature et Moralité. CH. CHABOT. Paris, 1897, pp. 287.

The moral object is one form of the beautiful. The good, true and useful,which are the contents of morality and religion; metaphysics and social authority, which supply the objective principles of obligation as well as sensory motives, and the practical reason, which supplies its subjective principles, are all manifestations of the good will which nature manifests.

Précis de Logique Évolutionniste: L'Entendement dans ses rapports avec le langage. PAUL REGNAUD. Paris, 1897, pp. 211.

The author, who is well known as a professor of Sanscrit and comparative grammar, who has published many works in his field, holds that language and reason are as closely related as physics and

chemistry, and each is needed to understand the other. Language is applied logic and logic is only a codification of the laws of language. This is seen in genus and species, the parts of speech, metaphors and tropes, categories, amphibolies, sophistries, definition, cause, proof, etc.

The Will to Believe, and Other Essays in Popular Philosophy. WILLIAM JAMES. Longmans, Green & Co., 1895, pp. 333.

Professor James has done a service to his many friends by reprinting with little change these earlier and later papers. They are The Will to Believe, Is Life Worth Living? The Sentiment of Rationality, Reflex-action and Theism, The Dilemma of Determinism, The Moral Philosophy and the Moral Life, Great Men and their Environment, The Importance of Individuals, On some Hegelisms, What Psychic Research has Accomplished. The author's charm of style makes him one of the best essayists who now write in our language.

La Religion de la Science et de L'Esprit. J. STRADA. Paris, 1897, pp. 405.

This, we believe, is about the thirty-fifth volume by the author since 1865. This is only the first volume on the scientific constitution of religion, and is a part of the "Philosophy of Methodic Impersonalism." Religion is the unity toward which science tends, and its identity with science is the *ultimum organon*. The only religion is that of facts simply described, hence it is all verity and means indefinite research. There is no need of messiahs, saviours, incarnation, etc. God is not word, but fact, and his revelations are impersonal.

Des Origines Épidémiques. DR. H. BOUCHER.

Bacteriology and microbism this author thinks will in the future be regarded as a fantastic dream. In a single year nearly 200,000 people died in France of tuberculosis, and the only prevention medicine suggests is not to spit in public places. Despite all the proud claims of the bacteriologist, typhus, lupus, diphtheria, rabies, and the rest show little decrease. Pathogenic organisms are not differentiated to make the basis for even the chief morbid entities. Many wrong diagnoses are cited, and the theories are often vague, incomplete and even contradictory. The author prefers a "rational" method which conceives health as a balance of external and internal influences, and speaks of morbific agents, infectious principles. We must rest content with such general principles till bacteriology is more advanced, and is cured of its exaggerations.

Étude Clinique du Dynamisme Psychique. DR. HENRI AIMÉ. Paris, 1897, pp. 258.

Dynamism is a function, and not, as was once thought, independent of organic conditions. It is a science joining psychology and physiology, but distinct from either. Starting from Brown-Sequard's and Exner's idea of a power of certain parts of the nervous system to augment or diminish the activity of other parts, and defining dynamism as an abnormal nervous activity which corresponds to no apparent organic alteration, which may be effaced by another therapeutic dynamism, especially by suggestion, the author proceeds to describe with considerable fullness seventy-four cases, mostly for Bernheim's hospital, illustrating his thesis. Etiology, diagnosis and prognosis are briefly treated and a good bibliography appended.

Analysis of the Sensations. ERNST MACH. Translated by C. M. Williams. Pp. xii-208; 37 cuts. The Open Court Publishing Co., Chicago, 1897. Price, $1.25.

It is pleasant to welcome this excellent and well-known little book in English dress. The German text has been enlarged for this edition by a special preface, a number of additional notes, two appendices and a full index.

E. C. S.

On the Time of Reflex Winking. DAVID P. MAYHEW. Journal of Experimental Medicine,I, 1897, pp. 35-47; 5 plates.

Exner's measurements of this time, made more than twenty years ago, have remained until now practically the only ones. His figures gave averages of 0.0662 and 0.0578 seconds, depending on the strength of the stimulus—an electric shock applied to the other eye-lid. The measurements of Mayhew, executed with apparatus better suited to mark the exact beginning of the lid movement and with a light blow on the face as a stimulus, give an average of 0.0420 from 450 measurements on sixteen subjects, male and female. It was found further that there were considerable individual differences (0.0351-0.0491), but that each individual was tolerably concordant with himself. Sex made no certain difference, nor did natural winks occurring just before those in response to the stimulus, but the time appeared to be somewhat shortened by apprehension of the blow.

E. C. S.

The Statement of Stella Maberly. A Novel. F. ANSTEY. London: T. Fisher Unwin, 1896, pp. 250.

Novels dealing with psychological subjects usually make tiresome reading to a psychologist, but in this story Mr. Anstey has succeeded in grasping the method of the paranoiac mind so well as to furnish an interesting book. It can appeal only to a limited class of readers, since alienists and psychologists alone can appreciate the thread of heredity hallucination and delusion on which the narrative is based. It is to be regretted that this is the case, because the laity are much in need of education regarding the dangers to be apprehended from this class, and a popular work of fiction might do something toward awakening a healthy interest in this important subject. The reasoning of Stella Maberly is directly in line with the methods of paranoiacs, and the tragic consummation is not an uncommon sequel to such mental processes. While it will prove to be nothing more than a gruesome tale to the average reader, it will certainly interest psychologists who are not averse to seeing the domain of abnormal psychology invaded by the novelist.

J. W. WALKER, M. D.

NOTES AND NEWS.

Experimental Psychology in England.

There are signs that England is at last falling into line with other nations in the matter of Experimental Psychology. At Cambridge two rooms in the Cavendish Laboratory have for some time past been devoted to psycho-physical work; and it is now rumored that a modest laboratory is to be set up on an independent footing, and a University Lecturership established with a salary of $250. Efforts are also being made by Professor Sully and others to give the science independent status at University College, London. Lectures upon special departments of psycho-physics have been delivered for two or three years. Rooms are now offered for temporary use by Professor Carey Foster of the physical department; a sum of $350 has been contributed for the purchase of instruments; and it is hoped that sufficient funds may be raised to secure the services of a trained demonstrator for at least one term of the academic year. Subscriptions may be sent to the Hampstead Branch of the London & Southwestern Bank (account Psychological Laboratory, University College).

While this news must be welcome to experimental psychologists the world over, it is to be hoped that English men of science will not rest satisfied with the success of so limited a program. An efficient laboratory requires nowadays an initial endowment of at least $2,000, and a yearly income of at least $500 more. Moreover, though psychology owes a great debt of gratitude to physiology and physics for the hospitable reception accorded to it in these two cases, there can be no doubt that such affiliation, however temporary, tends to weaken its claim to independence in the minds of the non-elect. This is abundantly shown by the results of the hospitality offered to homeless psychologists by the Physiological Laboratory at Oxford. Experimental psychology is not physiology; it cannot be taught by a physiologist; its methods and much of its apparatus are peculiar to itself. The beginnings at Cambridge and London must, therefore, be regarded as the thin end of a wedge whose potential broadening is as considerable as that of, say, physical chemistry or bacteriology. May it be driven deeply and persistently!

Psychology in the College Curriculum.

The above remarks call to mind a statement made by Professor Jastrow in a recent number of *Science* (June 4, 1897), to the effect that "psychology has a very unfortunate reputation in the mind of the college student, as a study peculiarly difficult, to be pursued by methods unusual and intricate." It would be interesting to

know, first, whether this statement really holds; and secondly, whether, in cases where it does hold, its truth is not referable to the fact that psychology is taught along old-fashioned lines. The writer's experience is that when stress is laid upon the performance of introspective exercises by the student, and the lecture series enlivened by experimental demonstration, psychology yields to but few studies in point of attractiveness, rivaling even subjects so intrinsically interesting as physiology; while its difficulty is no greater, and may be less, than that of some of the other old-established features of the college curriculum.

A Psycho-physical Vocabulary.

The following translations are additions to or substitutes for the renderings of German terms offered in this Journal, Vol. VII, pp. 78 ff.

Anfangsgefühl, initial feeling.
Endgefühl, terminal feeling.
Gebilde, formation.
Gegenfarbe, antagonistic color.
Gesammtvorstellung, aggregate idea.
Randcontrast, marginal contrast.
Schärfe, accuracy, acuteness.
Zwangs-, imperative.

The whole vocabulary should be compared with the glossary appended by Professor Wundt and Dr. Judd to the English translation of Wundt's *Grundriss*. It is noteworthy that Wundt has approved 'affective' as the equivalent of *Gemüths*. On the other hand, the rendering of *Gebilde* by 'compound' is not only inadequate, but in places becomes self-contradictory. The geological association of the term 'formation' seems to be, at least, harmless.

News.

Mr. J. F. Crawford has been appointed Demonstrator in Experimental Psychology at Princeton University.

Dr. W. B. Pillsbury, recently made Instructor in Psychology at Cornell University, has been called to the charge of the Department of Psychology in the University of Michigan.

Dr. G. Simmara has been nominated to the chair of Physiological Psychology in the Government School of Science, Madrid.

An International Congress of Neurology, Psychiatry, Medical Electricity and Hypnology will be held at Brussels from Sept. 14 to Sept. 19 of the present year. Address of the general secretary, Dr. Crocq *fils*, 27 Avenue Palmerston, Brussels.

A section will be devoted to Experimental Psychology at the forthcoming meeting of the British Association in Toronto. Notices of papers should be sent to Dr. A. Kirschmann, University of Toronto, Toronto, Can.

Books Received.

Aars. Die Autonomie der Moral mit besonderer Berücksichtigung der Morallehre Immanual Kants. Pp. 124. L. Voss, Hamburg u. Leipzig, 1896. Price, 3 m. Notice in No. 3, p. 423.

Basch. Essai Critique sur L'Esthétique de Kant. Pp. 623. F. Alcan, Paris, 1896. Price, 10 f.

BERGSON. Matière et Mémoire. Essai sur la Relation du Corps a L'Esprit. Pp. iii-280. Alcan, Paris, 1896. Price, 5 f. Notice in No. 3, p. 419.

BINET. L'Année Psychologique. Troisième Année. Librairie C. Reinwald, Paris, 1897. Pp. 825. Price, 15 f.

BROCHARD. De L'Erreur. Deuxième edition. Pp. 281. F. Alcan, Paris, 1897. Price, 5 f.

BRUNSCHVICG. La Modalité du Jugement. Pp. 246. F. Alcan, Paris, 1897. Price, 5 f.

CHABOT. Nature et Moralité. Pp. 290. F. Alcan, Paris, 1897. Price, 5 f.

DAURIAC. La Psychologie dans L'Opéra Francais (Auber, Rossini, Meyerbeer). Pp. xxiii-164. F. Alcan, Paris, 1897. Price, 2.50 f.

DESSOIR. Geschichte der neueren deutschen Psychologie. Zweite völlig umgearbeitete Auflage. Erster Halbband. Pp. 356. Carl Duncker, Berlin, N. W. S., 1897. Price, 8 m.

FERRIÈRE. La Cause Première d'après les Données Expérimentales. Pp. 462. F. Alcan, Paris, 1897. Price, 3.50 f.

GAMBLE, ELIZA BURT. The God-Idea of the Ancients, or Sex in Religion. Pp. v-339. G. P. Putnam's Sons, New York, 1897. Price, $2.25.

GAVARD. A Diplomat in London. Holt & Co., New York, 1897.

LADD. Philosophy of Knowledge. An Inquiry into the Nature. Limits and Validity of Human Cognitive Faculty. Pp. xv-614, Charles Scribner's Sons, New York, 1897. Price, $4.

LEDANTEC. Le Determinisme Biologique et la Personalité Consciente. Pp. 158. Alcan, Paris, 1897. Price, 2.50 f. Notice in No. 3, p. 418.

JODL. Lehrbuch der Psychologie. Pp. xxiv-767. J. Q. Cotta'sche Buchhandlung. Stuttgart, 1896. Notice in No. 3, p. 424.

MACH. Contributions to the Analysis of the Sensations. Translated by C. M. Williams. Pp. xii-208. Open Court Publishing Co., Chicago, 1897. Price, $1.25.

MOLL. Hypnotism. Fourth edition, revised and enlarged. Contemporary Science Series. Walter Scott, Ltd., London, 1897. Pp. xiv-448. Price, 3s. 6d.

MORGAN. Habit and Instinct. Pp. 350. Edward Arnold, London and New York, 1896. Price, $4.

NEUMARK. Die Freiheitslehre bei Kant und Schopenhauer. Pp. xii-90. L. Voss, Hamburg u. Leipzig, 1896. Price, 2 m. Notice in No. 3, p. 423.

PEARSON. The Chances of Death, and other Studies in Evolution. 2 vols. Edward Arnold, London and New York, 1897. Price, $8.

PEEK. The Chariot of the Flesh. Pp. 313. Longmans, Green & Co., New York, 1897. Price, $1.25.

RÉCÉJAC. Essai sur Fondements de la Connaissance Mystique. Pp. 306. Alcan, Paris, 1897. Price, 5 f. Notice in No. 3, p. 424.

REGNAUD. Précis de Logique Evolutionniste, l'Entendement dans ses Rapports avec le Langage. Pp. 215. F. Alcan, Paris, 1897. Price, 2.50 f.

RIBOT. L'Evolution des Idées Générales. Pp. 260. F. Alcan, Paris, 1897. Price, 5 f.

RIGOLAGE (Résumé par). La Sociologie par Auguste Comte. Pp. 472. F. Alcan, Paris, 1897. Price, 7.50 f.

SÉAILLES. Essai sur le Génie dans L'Art. Pp. xii-313. Alcan, Paris, 1897. Price, 5 f.

STRADE. La Religion de la Science et de L'Esprit Pur. Pp. 405. F. Alcan, Paris, 1897. Price, 7 f.

VIALLET. Je Pense, donc Je Sais. Introduction a la méthode cartésienne. Pp. 138. F. Alcan, Paris, 1897. Price, 2.50 f.

WETTERSTRAND. Hypnotism and its Application to Practical Medicine. Translated by Henrik G. Petersen, M. D. Pp. xvii-166. G. P. Putnam's Sons, New York, 1897. Price, $2.

WILDER. Neural Terms, International and National. (Reprinted from the *Journal of Comparative Neurology*, Vol. VI, December, 1896; issued February, 1897.) Pp. 137. Price, $1.

WUNDT. Outlines of Psychology. Translated with the coöperation of the author, by Charles Hubbard Judd, Ph. D. Pp. 342. Wilhelm Engelmann, Leipzig, 1897. Stechert, New York. Price, $1.75.

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ORIGINAL ARTICLES.

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(The names of those who have contributed original matter are printed in SMALL CAPITALS.)